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Formally Deriving the Third Newton's Law from a Pair of Nontrivial Assumptions in a Formal Axiomatic Theory "Sigma-V"

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Abstract

The article is devoted to hitherto never undertaken applying an almost unknown logically formalized axiomatic epistemology-and-axiology system called "Sigma-V" to the Third Newton's Law of mechanics. The author has continued investigating the extraordinary (paradigm-breaking) hypothesis of formal-axiological interpreting Newton's mathematical principles of natural philosophy and, thus, has arrived to *discrete mathematical modeling* a system of formal axiology of nature by extracting and systematical studying its *proper algebraic aspect*. Along with the proper *algebraic* machinery, the *axiomatic* (hypothetic-deductive) method is exploited in this investigation systematically. The research results are the followings. 1) The Third Newton's Law of mechanics has been modeled by a formal-axiological equation of two-valued algebraic system of metaphysics as formal axiology. (Precise defining the algebraic system is provided.) The formal-axiological equation has been established (and examined) in this algebraic system by accurate computing compositions of relevant evaluation-functions. Precise tabular definitions of the evaluation-functions are given. 2) The wonderful formula representing the Third Newton's Law (in the relevant physical interpretation of the formal theory Sigma-V) has been derived logically in Sigma-V from the presumption of a-priori-ness of knowledge. A precise axiomatic definition of the nontrivial notion "a-priori-ness of knowledge" is given. The formal derivation is implemented in strict accordance with the rigor standard of D. Hilbert's formalism; hence, checking the formal derivation submitted in this article is not a difficult task. With respect to proper theoretical physics, the formal inference is a *nontrivial scientific novelty* which has not been discussed and published elsewhere yet.

Keywords

Third Law of Newton's Mechanics, Logically Formalized Axiomatic Theory Σ-V, Two Valued Algebraic System of Metaphysics as Formal Axiology,

A-Priori Knowledge, Formal Derivation from Assumption

1. Introduction

The present article is an original attempt to apply the formalism (proclaimed and elaborated by D. Hilbert) to philosophical grounding some aspects of proper theoretical physics. In the well-known controversy between D. Hilbert and the "anti-formalists", for example, L. E. J. Brouwer [1] [2] and H. Poincaré [3] [4], I would like to side with Hilbert's famous ideal and program of/for axiomatizing and logical formalizing mathematical and physical theories [5]-[10]. In some significant aspects, the given paper follows that program, but in some other significant aspects, the paper deviates from that special interpretation of the program which has been suggested and developed by positivist-minded physicists and philosophers of science, for example, R. Carnap [11] [12] [13] [14] [15] and M. Schlick [16] [17] [18]. Today axiomatic method has been recognized as an effective one and has been used systematically in many qualitatively different spheres of human knowledge, for instance, in mathematics and logic [5] [6] [7] [8] [19] [20] [21] [22] [23], theoretical physics [9] [24] [25], economics [26], biology [27], and even theology [28]. I deliberately accept Hilbert's optimistic ideal of representing (modeling) content theories (loosely formulated in ambiguous natural language) by logically formalized axiomatic theories rigorously formulated in unambiguous artificial language (syntaxis and semantics of which are defined precisely). However, I do not agree with R. Carnap's and other positivists' proclamations that metaphysics and axiology are to be eliminated from proper physical theories completely [11]-[18] [29]. I believe that some precisely defined aspects of proper philosophy (especially, universal philosophical epistemology, universal philosophical ontology, universal formal logic, and universal formal axiology) are to be presented somehow in artificial language of well-done (sufficiently rich and heuristically significant) logically formalized axiomatic theories of physics.

Let us begin with characterizing the historical-philosophical background of the research presented in this article, its heuristically significant prerequisites and habitual associations. In times of I. Newton, proper theoretical physics had been called "natural philosophy", for example, I. Newton's famous treatise on physics had been titled "Mathematical Principles of Natural Philosophy" [30]. Certainly, Newton's works on mathematical grounding theoretical physics had produced a strong influence on I. Kant's discourse of metaphysics of nature. While writing relevant aspects of "Critique of Pure Reason" [31] and then "Prolegomena…" [32], Kant, had been keeping Newton's classical mechanics in mind and had made several references to it. While developing the doctrine of synthetical *pure a priori knowledge of necessarily universal principles of nature*, Kant had mentioned the first and the third Newton's laws of mechanics as concrete examples of such *pure a priori knowledge* of *strictly universal* principles of physics. To become quite convinced of this fact of history of philosophy of nature, let us look into the following citations from Kant's texts. He writes: "The science of *natural philosophy* (physics) contains in itself synthetic judgements *a priori*, as principles. I shall adduce two propositions. For instance, the proposition, "In all changes of the material world, the quantity of matter remains unchanged"; or that, "In all communication of motion, action and reaction must always be equal." In both of these, not only is the necessity, and therefore their origin *a priori* clear, but also that they are synthetical propositions" ([31], p. 18). Also, Kant writes: "As to the existence of pure natural science, or physics, perhaps many may still express doubts. But we have only to look at the different propositions which are commonly treated off at the commencement of proper (empirical) physical science—those, for example, relating to the permanence of the same quantity of matter, the *vis inertiae*, the equality of action and reaction, etc.—to the soon convinced that they form a science of pure physics (*physica pura*, or *rationalis*), which well deserves to be separately exposed as a special science, in its whole extent, whether that be great or confined" ([31], p. 19).

According to the above-cited statements by Kant, the *pure natural philosophy* (proper theoretical physics) is such a synthetical *a priori* knowledge of nature, which (knowledge) is *necessarily universal for* any physical experience and, consequently, is a *necessary condition for* all possible physical facts [31] [32] [33]. In particular, Kant has exemplified his concept "*pure a priori principle of natural philosophy*" by the famous law of inertia (the so-called Newton's First Law of mechanics originally formulated by Galileo Galilei [34] and then explicated and generalized by Descartes) [35]. Also, according to the above citations, Kant has instantiated his notion of "*pure a priori principle of rational physics*" by the *equivalence of action and reaction*, which equivalence is well-known today under the name "Newton's Third Law of Mechanics" [30].

With respect to a modernized representation of conjunction of Newton's and Kant's legacies relevant to physics at the level of today logic, methodology, and philosophy of science, the following interesting question arises. Suppose that there is an adequate logically formalized axiomatic system of philosophy (ontology, epistemology, etc.) quite applicable to *pure a priori knowledge of nature* (purely rational natural philosophy) as well. Is it possible, within that hypothetical logically formalized axiomatic system, to construct *formal deductive inferences* of the *pure a priori principles of rational physics* mentioned by Kant, from the assumption that knowledge is *a priori* one? For developing proper theoretical physics, this nontrivial question is very interesting (and, certainly, very difficult).

The formal deductive inference of the wonderful formula modeling the First Newton's Law of mechanics (the principle of inertia) in the logically formalized axiomatic epistemology-and-axiology system called "Sigma" (given the relevant physical interpretation of it) from the assumption of knowledge a-priori-ness has been already invented (constructed) and published in [36]. Is it possible to do "the same" with respect to the Third Newton's Law of mechanics? Can the

Third Law be *formally deductively derived* from the same assumption within the hypothetical logically formalized axiomatic theory? I guess that yes, in principle, it is possible. But the hypothesis has to be precisely formulated and well-grounded (it is to be *demonstrated deductively* at the level of its mathematical model). Therefore, below in the present article I am to undertake a systematical investigation targeted at explicating and proper theoretical grounding the guess by means of an appropriate mathematical apparatus. Namely, according to the indicated purpose, I am to invent (construct) a formal deductive inference of formula modeling the Third Law in a novel logically formalized axiomatic theory called "Σ-V" (under a relevant physical interpretation of this formal theory) from the assumption of a-priori-ness of knowledge.

Thus, the *main purpose of the research* presented in this article, is inventing (constructing) a formal deductive inference of such formula in Σ-V, which (formula) represents (under a relevant physical interpretation of Σ-V) the Third Law of Newton's mechanics, given that the assumption of a-priori-ness of knowledge is accepted. But, for successful realizing the manifestly formulated *main purpose of the research*, it is necessary to have quite adequate means, especially, an appropriate mathematical apparatus. Therefore, first of all, I have to realize the very important auxiliary purpose which is inventing (constructing) and precise defining a significantly new (hitherto never considered) logically formalized axiomatic epistemology-and-axiology system Σ-V. A successful realization of this indispensable auxiliary purpose is a necessary condition (prerequisite) for a successful realization of the main goal. This is so because, while investigating the hypothesis concerning the Third Newton's Law of mechanics, I have recognized that, with respect to this law of dynamics, Σ is not quite adequate (not rich enough); it is verisimilar that to cope with the task, one has to add to Σ some significantly new symbols (of the alphabet of object-language), new terms, novel formulae, definitions, and even some substantially new axioms. In the present article, the original outcome of the mentioned significant mutations in Σ is called "Sigma Vectored" or simply "Sigma-V". A precise definition of Sigma-V is to be given in the following section 2.1 of this paper. The promised realization of the main purpose of the article, namely, a formal deductive inference of the wonderful formula modeling the Third Newton's Law (in the relevant physical interpretation of Σ-V under the assumption of a-priori-ness of knowledge) is to be presented in the following section 3 of the given article. A realization of the above-formulated auxiliary purpose of the article is presented in the immediately following section.

2. Materials and Methods

2.1. A Hitherto Unknown Logically Formalized Axiomatic Theory "Sigma-V" Intentionally Constructed for Formal Axiomatic Grounding the Classical Theoretical Mechanics

Materials of the present article belong to physics and especially to that part of theoretical physics which deals with *necessarily universal* mathematical principles of natural philosophy (pure *a priori* knowledge of nature). As the present article is targeted at formal axiomatic grounding the third Newton's law within the philosophical rationalism doctrine accepting the assumption of pure a-priori-ness of rational knowledge of nature, in this part of the article, it is indispensable to give a precise definition of the mentioned assumption and of the formal axiomatic theory, to be exploited for realizing the target. The indirect but quite precise axiomatic definition of the assumption of a-priori-ness of knowledge is given below in this part of the paper, by means of precise defining the logically formalized axiomatic theory "Sigma-V".

In result of (1) adding the modality C ("It is *consistent* that…") to the set of *perfection*-modalities of the multimodal system Σ, and (2) significant *generalizing* the axiom-scheme AX-5 of Σ, a new system (named "Σ+C") has come into being. The axiomatic system Σ-V is a result of developing further the formal axiomatic *epistemology-and-axiology* theory Σ [37] and the formal axiomatic *epistemology-and-axiology* theory Σ+C [38].

To construct a perfectly exact definition of the formal axiomatic theory Σ-V, it is necessary to begin with manifestly giving precise definitions of the notions: "*alphabet* of object-language of Σ-V"; "*term* of Σ-V"; "*formula* of Σ-V"; "*axiom* of Σ-V". Strict definitions of these notions of Σ-V *look similar* to the definitions of corresponding notions of Σ and "Σ+C", which are already published (open access) in [37] and [38], respectively. Nevertheless, strictly speaking, in this article, it is quite indispensable to construct precise definitions of "*alphabet* of object-language of Σ-V", "*term* of Σ-V", "*formula* of Σ-V", and "*axiom* of Σ-V", in spite of the mentioned similarity, as *similarity is not logically equivalent to identity*; the relevant notions of Σ and "Σ+C" are not identical to the corresponding similar notions of Σ-V. Therefore, let us start precise formulating the definitions quite indispensable for perfect understanding this article in spite of the false impression (illusion) that they are repetitions of the already published statements. Let us begin with precise defining the notion "alphabet of object-language of formal theory Σ-V".

By definition, the alphabet of object-language of formal theory Σ-V contains all the signs which belong to the alphabet of object-language of formal theory Σ. But the conversion of this sentence is not true, as, in Σ-V, some important new symbols are added to the alphabet of Σ and to the alphabet of Σ+C. The outcome of these significant mutations (additions) is the following exact definition of the alphabet of object-language of Σ-V.

1) The lowercase Latin letters p, q, d (and these letters having lower number indexes) belong to the alphabet of object-language of Σ-V; these lowercase Latin letters are named "*propositional* letters". In the alphabet of object-language of Σ-V, *not all lowercase Latin letters are called propositional ones* because, according to the given definition, those lowercase Latin letters which belong to the set {g, b, e, n, x, y, z, t, f} do not belong to the set of *propositional* letters of object-language of Σ-V.

2) The habitual logic symbols ¬, ⊃, ↔, &, ∨ named, respectively, "classical

negation", "classical (or 'material') implication", "classical equivalence", "classical conjunction", "classical not-excluding disjunction" belong to the alphabet of object-language of Σ-V.

3) Elements of the set {□, K, A, E, S, T, F, P, D, C, G, W, O, B, U, J} are elements of the alphabet of object-language of Σ-V as well. They are named "modality symbols" in Σ-V.

4) The signs "→" and "←", called "vector symbols" or "arrows" ("left-right arrow" and "right-left one") belong to the alphabet of object-language of Σ-V. Such elements are quite novel; the "arrows" do not belong to the alphabets of object-languages of the already published and investigated formal theories Σ and Σ+C. The "vector symbols" belonging to the alphabet of Σ-V are *original* and very important ones for that theory. By the way, the "vector symbols" are not habitual (and even very unusual, odd) for object-languages of formal theories based on classical symbolic logic. Thus, even at the level of its alphabet, Σ-V differs much from the theories Σ and Σ+C.

5) The lowercase Latin letters *x*, *y*, *z* (and also these letters having lower number indexes) belong to the alphabet of object-language of Σ-V. Such and only such letters are named "*axiological variables*" in Σ-V.

6) The lowercase Latin letters "g" and "b" named "*axiological constants*" also are elements of the alphabet of object-language of Σ-V.

7) The capital Latin letters having number indexes—E^1, C^1, K^1, K^2, E^2, C^2, C_j^n, B_i^n, D_m^n, A_k^n, are elements of the alphabet of object-language of Σ-V (such capital Latin letters are named "*axiological-value-functional symbols*"). Here the upper number index *n* informs that the indexed axiological-value-functional symbol is *n*-placed one. Absence of the upper number index indicates that the value-functional symbol is determined by only one axiological variable. The axiological-value-functional symbols may possess no lower number index. But, if value-functional symbols possess lower number indexes, then, if these indexes are different, then the indexed functional symbols are different ones.

8) The signs "(" and ")" named "round brackets" are elements of the alphabet of object-language of Σ-V as well. These auxiliary signs are utilized in the present article as usually in symbolic logic, namely, as pure technical symbols.

9) The signs "[" and "]" ("square brackets") are elements of the alphabet of object-language of Σ-V also. However, it is worth emphasizing here that in contrast to the "round brackets", in Σ-V, the "square brackets" are used not as the habitual pure technical symbols, but as *ontologically meaningful* signs. Such nonstandard using the "square brackets" is psychologically unexpected (unhabitual) one. In relation to natural language psychology, square brackets and round ones seem identical as very often in natural language they are used as synonyms. But in the object language of Σ-V, the two kinds of brackets possess *significantly different* meanings (play substantially different roles): usage of round brackets is purely technical (auxiliary) one, while square-bracketing possesses an *ontological* meaning. The *ontological* meaning of square-bracketing is defined below in that part of the present paper which is devoted to *semantics* of object-language

of Σ-V. Nevertheless, even at the level of syntaxis of the artificial object language of Σ-V, square brackets *play a substantial role* in the precise definition of the concept "formula of Σ-V". (This definition is to be given below in this section of the article.) Moreover, square-bracketing *plays a substantial role* in the precise formulations of some axiom-schemes of Σ-V" (which formulation is to be given below also in this section of the article).

10) An unhabitual artificial symbol "=+=" named "*formal-axiological equivalence*" is an element of the alphabet of object-language of Σ-V. The odd symbol "=+=" *plays a substantial role* in the precise definition of the concept "formula of Σ-V and also in the precise formulations of some axiom-schemes of Σ-V.

11) The habitual symbols "-" (negative number sign called "minus") and "=" (equality of numbers) from the language of arithmetic are elements of the alphabet of object-language of Σ-V.

12) The habitual symbol "/" also belongs to the alphabet of object-language of Σ-V, although, in Σ-V, this quite habitual symbol is used in a quite unexpected (unhabitual) special meaning (to be defined precisely below while formulating semantics of Σ-V).

13) A sign is an element of the alphabet of object-language of Σ-V, if and only if the sign belongs to this alphabet due to the above-formulated items 1) - 12) of the given definition.

Any finite chain (queue) of symbols is named "an *expression* of the object-language of Σ-V", then and only then, when that chain contains such and only such signs which are elements of the alphabet of object-language of Σ-V.

A precise definition of the concept "*term* of Σ-V" is the following:

1) The above-mentioned *axiological variables* (see the definition of alphabet of Σ-V) are terms of Σ-V.

2) The above-mentioned *axiological constants* (see the definition of alphabet of Σ-V) are terms of Σ-V.

3) If Φ_k^n is an *n-placed axiological- value-functional symbol* (see the definition of alphabet of Σ-V), and $t_i, \ldots, t_n$ are *terms* of Σ-V, then $\Phi_k^n t_i, \ldots, t_n$ is a term of Σ-V. (It is worth noting here that signs $t_i, \ldots, t_n$ belong to the meta-language; because they denote *any* terms of Σ-V; the analogous note is worth making with respect to the sign Φ_k^n belonging to the meta-language as well.)

4) If t is a term of Σ-V, then the expressions $\overleftarrow{t}$, ®$\overleftarrow{t}$, and $\vec{t}$ are terms of Σ-V.

5) If t_k is a term of Σ-V, then the expression $/t_k/$ is a term of Σ-V.

6) If t_k is a term of Σ-V, then the expression $-/t_k/$ is a term of Σ-V.

7) An expression of the object-language of Σ-V is a term of Σ-V, then and only then, when it is so due to the above-formulated items 1) – 6) of the given definition.

Thus, the *syntaxis* aspect of the abstract notion "*term* of Σ-V" is quite fixed. Now we are to move to constructing exact definition of the *syntaxis* aspect of the abstract notion "*formula* of Σ-V". To perform this move, let us accept the convention that in the given article, lowercase Greek letters α, β, and ω (belonging to meta-language) denote *any* formulae of Σ-V. Keeping this convention in mind, it is possible to give the following precise definition of the notion "formula of Σ-V".

1) All the propositional letters belong to the set of formulae of Σ-V.

2) When α and β are formulae of Σ-V, then all the expressions of the object-language of Σ-V, which (expressions) have forms ¬α, (α ↔ β), (α ⊃ β), (α ∨ β), (α & β), belong to the set of formulae of Σ-V as well.

3) When t_i and t_k are terms of Σ-V, then (t_i =+= t_k) is a formula of Σ-V.

4) When t_i and t_k are terms of Σ-V, then (t_i =+= ® t_k) is a formula of Σ-V.

5) When t_i and t_k are terms of Σ-V, then (/t_i/ = /t_k/) is a formula of Σ-V.

6) When t_i and t_k are terms of Σ-V, then (/t_i/ = -/t_k/) is a formula of Σ-V.

7) When t_i is a term of Σ-V, then [t_i] is a formula of Σ-V.

8) When α is a formula of Σ-V, and the symbol Ψ (belonging to the meta-language) denotes any modality symbol from the set of {□, K, A, E, S, T, F, P, D, C, G, W, O, B, U, J}, then any expression of object-language of Σ having the form Ψα, is a formula of Σ-V also. It is worth noting here, that, strictly speaking, the expression Ψα (belonging to the meta-language) is not a formula of Σ-V, but a scheme of formulae of Σ-V.

9) Chains of symbols from the alphabet of object-language of Σ-V are formulae of Σ-V, if and only if it is so due to the items 1) - 8) of the given definition.

In this part of the article which (part) is reduced intentionally to *syntaxis* of object-language of *multimodal* formal theory Σ-V, the set of modality symbols {□, K, E, A, S, T, F, P, D, C, G, W, O, B, U, J} is nothing but a set of very short *names*. The symbol □ is a name for the alethic modality "it is *necessary* that …". The symbols K, E, A, S, T, F, P, D, C, respectively, are names of/for the modal expressions "agent *Knows* that…", "agent *Empirically* (*a-posteriori*) *knows* that…", "agent *A-priori knows* that…", "under some concrete conditions in some definite time-and-space, an agent has a *Sensation*, *i.e. verification by feeling* (either immediately or by means of mediating tools), that…", "it is *True* that…", "agent has *Faith* that… (or agent believes that…)", "it is *Provable* in a consistent theory that…", "there is *an algorithm for Deciding* that… (hence, a machine could be constructed for such Deciding)", "it is *Consistent* that…".

The symbols G, W, O, B, U, J, respectively, are names of/for the modal expressions "it is *Good* (morally perfect) that…", "it is *Wicked* (morally bad, imperfect) that…", "it is *Obligatory* (mandatory, compulsory) that…", "it is *Beautiful* (aesthetically perfect) that…", "it is *Useful* (helpful, valuable, gainful, rewarding) that…", "it is a *Joy* (delight, happiness, pleasure) that…". In the present section of the article, pure syntaxis meanings of the modal symbols are defined quite precisely (although not manifestly but indirectly) by the below-given schemes of own (proper) axioms of multimodal formal philosophy (epistemology-and-axiology) system Σ-V which axioms are added to the ones of classical logic of propositions.

Thus, proper formal logic axioms and formal logic inference rules of Σ, Σ+C, and Σ-V are the ones of classical sentential logic calculus. Schemes of axioms and inference-rules of the classical propositional logic are applicable to all formulae of these three multimodal theories. Hence, the proper logic foundations of Σ, Σ+C, and Σ-V are identical but the mentioned logically formalized axiomatic systems based on these identical logic foundations are substantially different. It seems that, corresponding definitions of Σ, Σ+C, and Σ-V are identical, but strictly speaking, it only seems so. The formal theories Σ, Σ+C, and Σ-V have different alphabets of their object-languages, different sets of expressions, different sets of terms, different sets of formulae, different sets of definitions, different sets of axioms, and, finally, different sets of theorems.

In the given section of the article, exactly *syntax* meanings of all the modality symbols and of all the other special signs included into the alphabet of object language of Σ-V are defined precisely by the following list of schemes of proper philosophical (epistemological and axiological) axioms of Σ-V. (Certainly, such *axiomatic* definition of proper epistemology-and-axiology notions is *not manifest* one, but, nevertheless, it is *quite precise* one.) If α, β, ω are any formulae of Σ-V, then any such and only such expressions of the object language of Σ-V, which have the following forms, are *proper axioms* of Σ-V.

AX-1: $A\alpha \supset (\Box\beta \supset \beta)$.

AX-2: $A\alpha \supset (\Box(\omega \supset \beta) \supset (\Box\omega \supset \Box\beta))$.

AX-3: $A\alpha \leftrightarrow (K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta)))$.

AX-4: $E\alpha \leftrightarrow (K\alpha \,\&\, (\Diamond\neg\alpha \vee \Diamond S\alpha \vee \neg\Box(\beta \leftrightarrow \Omega\beta)))$.

AX-5: $\Omega\alpha \supset \Diamond\alpha$. (This is a substantial *multimodal generalization* of "Kant principle" combining the deontic and the alethic modalities: $O\alpha \supset \Diamond\alpha$.)

AX-6: $(\Box\beta \,\&\, \Box\Omega\beta) \supset \beta$. (This is a substantial *multimodal generalization* of the famous formula $(\Box\beta \supset \beta)$ underivable in Σ-V. Concerning the underivability of $(\Box\beta \supset \beta)$, see [39].)

AX-7: $(t_i =+= t_k) \leftrightarrow (G[t_i] \leftrightarrow G[t_k])$.

AX-8: $(t_i =+= g) \supset \Box G[t_i]$.

AX-9: $(t_i =+= b) \supset \Box W[t_i]$.

AX-10: $(G\alpha \supset \neg W\alpha)$. See the famous monograph by A.A. Ivin [40].

AX-11: $(W\alpha \supset \neg G\alpha)$. See A.A. Ivin's wonderful book [40].

AX-12: $A\alpha \supset ((\Phi\copyright xy =+= \Phi\copyright yx) \leftrightarrow ([\overleftarrow{\underleftarrow{\Phi\copyright xy}}] \leftrightarrow [\overleftarrow{\underleftarrow{\circledR\Phi\copyright yx}}]))$.

AX-13: $A\alpha \supset ((\Phi\copyright xy =+= \Phi\copyright yx) \leftrightarrow (/\Phi\copyright xy/ = -/\Phi\copyright yx/))$.

Definition DF-1: $((\circledR\overleftarrow{t_i} =+= \overrightarrow{t_i}) \,\&\, (\circledR\overrightarrow{t_i} =+= \overleftarrow{t_i}))$, where t_i is a term of Σ-V.

Definition DF-2: when ω is a formula of Σ-V, then ◊ω is a *name* of/for $\neg\Box\neg\omega$.

In AX-3, AX-4, AX-5, and AX-6, the symbol Ω (belonging to the meta-language) stands only for a (any) "*perfection* modality". Not all the above-mentioned modalities are called "*perfection* ones". The set Δ of signs denoting *perfection*-modalities (or simply, "perfections") is the following {K, D, F, C, P, J, T, B, G, U, O, □}. Obviously, Δ is only a subset of the set of all signs denoting modalities taken into an account in this article. For instance, W and ◊ are names of/for modalities which are not perfections.

In AX-12 and AX-13, the symbol Φ (belonging to the meta-language) stands for any noncommutative *binary* operationof two-valued Boolean algebra (of formal axiology, for example), and the symbol © (also belonging to the meta-language) stands for any such *unary* function,a value of which is the*inversion* of value of its argument.

Certainly, the above-given exact *syntactic* definitions are *semantically meaningless*; this is not a contingent omission by negligence but such a deliberately accepted scientific abstraction which is quite reasonable within an adequately defined domain. Therefore, now, to make the article perfectly meaningful one, it is indispensable to move directly to *semantics* of the language of Σ-V.

2.2. Hitherto Never Considered Semantics of/for the Above-Defined Syntaxis of Object-Language of Formal Axiomatic Theory "Sigma-V"

Above in paragraph 2.1 of this paper, the definition of Σ-V has been given in the purely syntactic manner; the formulation of Σ-V has been intentionally deprived of its concrete contents by means of the relevant scientific abstraction. The multimodal axiomatic theory Σ-V has been defined and discussed as exactly formal one. Below in this paragraph of the paper, I depart from pure syntaxis of artificial language of Σ-V to its semantics.

The artificial language of Σ-V contains the well-known logic symbols of classical mathematical logic. There is no necessity to give definitions of semantic meanings of these habitual logic symbols as their semantic meanings are already defined precisely in corresponding handbooks on mathematical logic. Semantic meanings of the *propositional variables* (represented in Σ-V by the lowercase Latin letters "d", "q", "p", and by the same letters possessing lower number indexes) are perfectly defined in corresponding handbooks on classical propositional logic as well. However, it is indispensable to give definitions of semantic meanings of the unhabitual signs (sometimes even strange complex ones) belonging to the artificial language of Σ-V.

Definition of semantic meanings is definition of an *interpretation-function*. For defining the interpretation-function it is necessary to define 1) a set called "domain (or realm) of interpretation" (let the letter M be used for denoting the interpretation domain) and 2) an evaluation-maker called "valuator" *V*. By definition, the set M (which is necessary for any standard interpretation of Σ-V), is a set, every element of which possesses: 1) one and only one *axiological value* from the set {good, bad}; 2) one and only one *ontological value* from the set {exists, not-exists}.

The *axiological variables* (z, x, y, z_i, x_k, y_m) take their values from the set M.

The *axiological constants* "b" and "g" mean the *values* "bad" and "good", respectively.

Certainly, any concrete valuation necessarily implies existence of a concrete valuator (interpreter). Making an evaluation of an element from the interpretation-domain M by quite a definite (fixed) valuator V is attaching an *axiological value* (good or bad) to the element. The valuator V is either any individual or any collective—it does not matter. Obviously, changing V may result in changing some valuations (relative ones), nevertheless, no change of valuator can change the set of laws of the algebraic system of formal axiology as these laws are not relative but absolute evaluations. By definition, the laws of two-valued algebra of formal axiology are such and only such *constant evaluation-functions* which possess the value g (good) under any possible combination of axiological values of their axiological variables. Certainly, V is a variable. It takes its values from the set of various valuators. However, if an interpretation of Σ-V is well-defined, then the value of the variable V is well-defined also. Changing the value of V is changing the interpretation.

In the present article, "e" and "n" stand for "… exists" and "… not-exists", respectively. The signs "e" and "n" are named "*ontological constants*". By definition, in a standard interpretation of Σ-V, one and only one element of the set {{g, e}, {g, n}, {b, e}, {b, n}} corresponds to every element of M. The signs "e" and "n" belong to the meta-language. By definition of the alphabet of object-language of Σ-V, "e" and "n" do not belong to the object-language. Nevertheless, "e" and "n" are *indirectly* represented at the level of object-language of Σ-V by means of *square-bracketing*: "t_i exists" is represented by [t_i]; "t_i does not exist" is represented by ¬[t_i]. This means that square-bracketing is a significant part of exact defining formal-axiological-and-ontological semantics of Σ-V.

N-placed terms of Σ-V are interpreted as *n*-placed evaluation-functions defined on the set M. The concept "unary (or one-placed) evaluation-function" is instantiated by the following **Table 1** and **Table 2**. Here, it is worth recalling that the upper index 1 standing immediately after a capital letter means that the indexed letter denotes a unary evaluation-function. A Difference of lower number-indexes means difference of the corresponding signs. For instance, in **Table 1**, F_1^1x and F_2^1x are different signs. In **Table 2**, A_3^1x and A_4^1x are different signs as well. Symbols may have no lower number-indexes, for example, M^1x. If a symbol does not have a lower number-index, then this symbol is different from the same symbol having a lower number-index. For example, M^1x and M_1^1x are different symbols. Also, it is worth recalling here that in all the below-located tables, symbols "g" and "b" stand for evaluations "good" and "bad", respectively.

Table 1. Defining the unary evaluation-functions.

x	M^1x	B_1^1x	N_1^1x	Z_1^1x	D_1^1x	G_1^1x	W_1^1x	F_1^1x	F_2^1x	F_3^1x	F_4^1x
g	b	g	b	b	g	g	b	b	g	b	g
b	g	b	g	b	g	g	b	g	b	b	g

Table 2. Defining the one-placed evaluation-functions.

x	A_1^1x	A_2^1x	A_3^1x	A_4^1x	M_1^1x	M_2^1x	M_3^1x	M_4^1x	P_1^1x	H_1^1x
g	b	g	b	g	b	b	g	g	g	b
b	g	b	b	g	b	g	b	g	b	g

In Table 1, the one-placed term M^1x is interpreted as *unary evaluation-function* "*movement, change of* (what, whom) *x*"; the one-placed term $B_1{}^1x$ is interpreted as *unary evaluation-function* "*being of* (what, whom) *x*"; the term $N_1{}^1x$ is interpreted as *unary evaluation-function* "*non-being of* (what, whom) *x*". $Z_1{}^1x$—"*absolute non-being of* (what, whom) *x*". $D_1{}^1x$—"*absolute being of* (what, whom) *x*". $G_1{}^1x$—"*absolute goodness of* (what, whom) *x*", or "*absolute good* (what, who) *x*". $W_1{}^1x$—"*x*'s being *absolute evil*", or "*absolute bad, wicked* (what, who) *x*". $F_1{}^1x$—"*force* (*power*) *over* (what, whom) *x*", or "applying force to *x*". $F_2{}^1x$—"*force* (*power*) *of* (what, whom) *x*", or "*x*'s power (force)", or "applying force by (what, whom) *x*". $F_3{}^1x$—"applying force (power) to *absolute good x*", *i.e.* violence against the absolute virtue *x*". $F_4{}^1x$—"force (power) of *absolute good x*", or "applying force (using coercion) by *absolutely good x*".

Now let us introduce the one-placed evaluation-functions represented by terms $A_1{}^1x$, $A_2{}^1x$, …, etc., which are relevant to the theme of the present article, by means of the following *glossary* for the above-located Table 2. The *glossary* introduces the functions one by one, namely, in Table 2, the one-placed term $A_1{}^1x$ is interpreted as unary evaluation-function "*action on* (what, whom) *x*". The term $A_2{}^1x$ is interpreted as one-placed evaluation-function "*action of* (what, whom) *x*", or "*action by* (what, whom) *x*", or "*x*'s *action*". $A_3{}^1x$—"*action on absolute good* of *x*". $A_4{}^1x$—"*action of* (what, whom) *absolute good of x*", or "*action by absolute good of x*". $M_1{}^1x$—"*primeval matter*, (*or prime matter*, *or materia prima*) *of* (what, whom) *x*". $M_2{}^1x$—"*matter*, *materialness of* (what, whom) *x*", or "*x*'s *being a material*". $M_3{}^1x$—"*matter*, *material for* (what, whom) *x*", or "*being a material for x*". $M_4{}^1x$—"*being a material* (*matter*) *for absolute good x*". $P_1{}^1x$—"*positive evaluation of* (what, whom) *x*". $H_1{}^1x$—"*negative evaluation of* (what, whom) *x*". The one-placed evaluation-functions, which are formal-axiological meanings of the symbols introduced by the glossary, are defined precisely by the above-located Table 2.

The concept "*two-placed evaluation-function*" is exemplified by the following Table 3. (In this article, the upper index 2 standing immediately after a capital letter informs that this letter denotes *a two-placed function*.)

In Table 3, the two-placed term S^2xy is interpreted as two-placed evaluation-function "*separation*, *divorcement* between *x* and *y*. The term K^2xy is interpreted as two-placed evaluation-function "*uniting x* and *y*", or "*being* of *x* and *y* together", or "*being of both x* and *y*". N^2xy is interpreted as binary evaluation-function "*realizing neither x nor y*". F^2xy—evaluation-function "*force of action of y applied to x*", or "*force of y*'*s action on x*". A^2xy—"*y*'*s action on x*". V^2xy—"*y*'*s violence on* (*over*) *x*". W^2xy—"*mutual action* of *x* and *y on each other*, or "*interaction* between *x* and *y*". C^2xy—"*contradiction* of *y to* (*or with*) *x*". Z^2xy—"*mutual contradiction* of *x* and *y to* (*or with*) *each other*, or "*contradiction* between *x* and *y*". E^2xy—"*equivalence* (*identity of values*) of *x* and *y*". For additional instantiations of the notion "*two-placed evaluation-function*" see [36] [37] [38] [39] [41] [42] [43].

Table 3. Defining the two-placed evaluation-functions.

x	*y*	S^2xy	K^2xy	N^2xy	F^2xy	A^2xy	V^2xy	W^2xy	C^2xy	Z^2xy	E^2xy
g	g	b	g	b	b	b	b	b	b	b	g
g	b	g	b	b	b	b	b	b	b	b	b
b	g	g	b	b	g	g	g	b	g	b	b
b	b	g	b	g	b	b	b	b	b	b	g

For excluding possible misunderstandings, it is relevant here to emphasize that in a standard interpretation of Σ-V, the signs F_1^1x, A_2^1x, M_3^1x, S^2xy, W^2xy, Z^2xy stand not for predicates but for *evaluation-functions*. If a standard interpretation of Σ-V is given, then such expressions of the object-language of Σ-V, which possess forms (t_i=+=b), (t_i=+=g), (t_i=+=t_k), represent *predicates* in Σ-V.

By definition of semantics of Σ-V, if t_i is a term of Σ-V, then, being interpreted, such a formula of Σ-V, which possesses the form [t_i], represents an *either true or false proposition* "t_i exists". According to the definition, a formula [t_i] is true in an interpretation, if and only if t_i possesses the *ontological value* "e (exists)" in that interpretation. According to the definition, a formula [t_i] is false in an interpretation of Σ-V, if and only if t_i possesses the ontological value "n (not-exists)" in that interpretation.

By definition of semantics of Σ-V, in a standard interpretation of Σ-V, a formula having the form (t_i=+=t_k) represents an either true or false proposition possessing the form "t_i is *formally-axiologically equivalent* to t_k"; this proposition is true, when and only when the terms t_i and t_k have identical *axiological values* (from the set {good, bad}) under any possible combination of *axiological values* of their *axiological variables*.

By definition of semantics of Σ-V, in a standard interpretation of Σ-V, a formula having the form (t_i=+=b) represents an either true or false proposition possessing the form "t_i is a *formal-axiological contradiction*" (or "t_i is *formally-axiologically, or absolutely bad*"); this proposition is true, when and only when (in the given interpretation) the term t_i has the value "bad" under any possible combination of axiological values of the axiological variables.

By definition of semantics of Σ-V, in a standard interpretation of Σ-V, a formula having the form (t_i=+=g) represents an either true or false proposition possessing the form "t_i is a *formal-axiological law*" (or "t_i is *formally-axiologically, or absolutely good*"); this proposition is true when and only when (in the given interpretation) the term t_i has the *value* "good" under any possible combination of the values of axiological variables.

Concerning the above-provided definition of sematic meaning of (t_i=+=t_k) in Σ-V, it is worth keeping in mind that, the natural-language words "imply", "entails", "is", "means", "equivalence" are homonyms; each of these words has not the only meaning. Along with possessing the well-known formal logic meanings, in some special kinds of situations, the same words of natural language may mean the above-defined *formal-axiological-equivalence* relation "=+=". The indicated ambiguity of natural language must be neutralized; therefore, the formal-logical and the formal-axiological meanings of the mentioned homonyms must be separated systematically; otherwise strong linguistic illusions of logical paradoxes can appear.

Due to the definition of semantics of Σ-V, one can recognize that the algebraic system of formal axiology is a *theory-of-relativity* of evaluations; in the theory-of-relativity, the *formal-axiological laws* of that algebraic system (which are nothing but positive-constant-evaluation-functions) are *invariants* in relation to all possible transformations of valuator V.

Thus, in spite of the indisputable (quite obvious) fact that relativity and flexibility of empirical evaluations do exist, the necessarily universal and immutable laws of relativity of evaluations (which laws are evaluation-*invariants*) do exist as well [37] [41] [42].

One of very important qualitative differences between Σ and Σ-V is existence of expressions having the forms "$/t_i/$", "$/t_k/$", "$-/t_k/$", "$/\overleftarrow{t_i}/$", "$/\overleftarrow{t_k}/$", "$-/\overleftarrow{t_k}/$", "$/t_i/ = /t_k/$", "$/\overleftarrow{t_i}/ = /\overleftarrow{t_k}/$", "$/\overleftarrow{t_i}/ = -/\overleftarrow{t_k}/$", in the object-language of Σ-V. Concerning the expressions having such forms, here it is indispensable to inform the readers that, by definition of semantics of object-language of Σ-V, the expression "/…/" means a "*quantity magnitude*" of "…"; the expression "-/…/" means a "*negative quantity magnitude*" of "…".

Also, here it is indispensable to inform the readers that, according to the definition of semantics of object-language of Σ-V, in a standard interpretation of this formal theory, the symbol "=" means the binary relation of "*identity of quantity magnitudes*", hence, "$/t_i/ = /t_k/$" means "*identity of quantity magnitudes* of t_i and t_k". Thus, if a concrete standard interpretation is given, then the (interpreted) formula ($/\overleftarrow{t_i}/ = -/\overleftarrow{t_k}/$) represents a predicate (which obtains quite a definite truth-value in the given concrete interpretation). It is presumed here that the syntaxis and semantic meanings of those signs from the alphabet of object-language of Σ-V, which are signs from the alphabet of formal arithmetic (for instance, the symbol "="), are already defined precisely in arithmetic [44], therefore, it is not necessary to define them again in the present article manifestly. Making the reference to E. Mendelson's excellent handbook "Introduction to Mathematical Logic" [44] is quite sufficing. Thus, although the relevant arithmetic axioms (namely, those which define meanings of the arithmetic signs "=" and "-") are not manifestly mentioned in the above-given definition of Σ-V, they are presumed as also belonging to Σ-V.

The main qualitative difference between Σ and Σ-V is existence of *vector* symbols "→" and "←" in the richer alphabet of object-language of Σ-V. The theories Σ and Σ+C deal with *scalar* evaluation-functions exclusively, while the significantly more general and more rich theory Σ-V deals with both *scalar* and *vector* ones. This is so because, by definition of semantics of Σ-V, the evaluation-functions are either *vectored* or *not-vectored*. (The not-vectored ones are called scalar ones.) The scalar-evaluation-functions are exemplified above by **Tables 1-3**. According to the definition of semantics of object-language of Σ-V, the symbol $\overleftarrow{t_i}$ means a *vectored* evaluation-function, which is nothing but a conjunction (union) of the corresponding scalar-evaluation-function (t_i) and the definite vector (direction) "←" of the evaluation. Sometimes, either an evaluation does not have a vector at all, or the vector of evaluating is not essential and may be ignored by means of abstraction. But, sometimes, the vector is essential (important) and ignoring it is a blunder (the abstraction is not acceptable). In this special case (which is very important one, sometimes), the sign "$\overleftarrow{t_i}$" means the vector (direction) of "t_i"; while the sign "$®\overleftarrow{t_i}$" means the *directly opposite* vector of "t_i". Certainly, from the psychological viewpoint, the hitherto unknown discourse of vectored evaluation-functions is queer one; it is a challenge for the habitual paradigm in the humanities to which theoretical philosophy belongs somehow (at least partly). Nonetheless, in modern theoretical physics (which could be considered as a necessarily mathematized kind of proper theoretical philosophy of nature), a discourse of vectored functions is not queer but quite habitual, meaningful and disputable. In the present article, I am to apply the precedent made in modern theoretical physics to epistemic decision-making in the *essentially analogous* situation of proper philosophical (formal-axiological) discourse of classical (Newton's) mechanics. Surprising results of such unexpected applying are presented in the immediately following paragraph.

3. Results

The following finite succession of formulae and schemes of formulae is a realization of the main purpose of this paper, namely, a *formal deductive derivation* (in Σ-V) of such a formula which represents Newton's Third Law, if a relevant physical interpretation of Σ-V is given. This formal logical derivation essentially depends from the two nontrivial *assumptions* manifestly included into the following finite succession. The first one is the assumption of a-priori-ness of knowledge. It is modeled in Σ-V by Aα. The second assumption essentially exploited in the following logical derivation is the *formal-axiological analog* of the Third Law of mechanics. The *formal-axiological analog* of that law is modeled in Σ-V by (F^2M^1xy =+= F^2M^1yx). This *formal-axiological equation* of two-valued algebraic system of metaphysics as formal axiology has been already recognized as such, and published in [42]. But the following quite a new formal deductive inference from the pair of nontrivial assumptions has not been constructed and published hitherto.

1) $A\alpha \supset \left(\left(\Phi©xy =+= \Phi©yx \right) \leftrightarrow \left(\left[\overleftarrow{\Phi©xy} \right] \leftrightarrow \left[®\overleftarrow{\Phi©yx} \right] \right) \right)$: AX-12.

2) $A\alpha \supset \left(\left(F^2M^1xy =+= F^2M^1yx \right) \leftrightarrow \left(\left[\overleftarrow{F^2M^1xy} \right] \leftrightarrow \left[®\overleftarrow{F^2M^1yx} \right] \right) \right)$: from 1, by substitution of F^2 for Φ, and substitution of M^1 for ©.

3) $A\alpha \supset \left(\left(\Phi©xy =+= \Phi©yx \right) \leftrightarrow \left(/\overleftarrow{\Phi©xy}/ = -/\overleftarrow{\Phi©yx}/ \right) \right)$: AX-13.

4) $A\alpha \supset \left(\left(F^2M^1xy =+= F^2M^1yx \right) \leftrightarrow \left(/\overleftarrow{F^2M^1xy}/ = -/\overleftarrow{F^2M^1yx}/ \right) \right)$: from 3, by substitution: of F^2 for Φ, and substitution of M^1 for ©.

5) Aα: the *assumption* of a-priori-ness of knowledge.

6) $\left(\left(F^2M^1xy =+= F^2M^1yx\right) \leftrightarrow \left(\left[\overleftarrow{F^2M^1xy}\right] \leftrightarrow \left[\overleftarrow{®F^2M^1yx}\right]\right)\right)$: from 2 and 5 by *modus ponens*.

7) $\left(\left(F^2M^1xy =+= F^2M^1yx\right) \leftrightarrow \left(/\overleftarrow{F^2M^1xy}/ = -/\overleftarrow{F^2M^1yx}/\right)\right)$: from 4 and 5 by *modus ponens*.

8) $\left(\left(F^2M^1xy =+= F^2M^1yx\right) \supset \left(\left[\overleftarrow{F^2M^1xy}\right] \leftrightarrow \left[\overleftarrow{®F^2M^1yx}\right]\right)\right)$: from 6 by the logic derivation rule called "elimination of $\leftrightarrow$".

9) $\left(\left(F^2M^1xy =+= F^2M^1yx\right) \supset \left(/\overleftarrow{F^2M^1xy}/ = -/\overleftarrow{F^2M^1yx}/\right)\right)$: from 7 by the logic derivation rule called "elimination of $\leftrightarrow$".

10) $\left(F^2M^1xy =+= F^2M^1yx\right)$: the *assumption*.

11) $\left(\left[\overleftarrow{F^2M^1xy}\right] \leftrightarrow \left[\overleftarrow{®F^2M^1yx}\right]\right)$:from 8 and 10 by *modus ponens*.

12) $\left(/\overleftarrow{F^2M^1xy}/ = -/\overleftarrow{F^2M^1yx}/\right)$: from 9 and 10 by *modus ponens*.

13) $A\alpha, \left(F^2M^1xy =+= F^2M^1yx\right) \vdash \left(\left[\overleftarrow{F^2M^1xy}\right] \leftrightarrow \left[\overleftarrow{®F^2M^1yx}\right]\right)$:by the succession 1 - 11. (Here, "…$\vdash$…" means "from…it is logically derivable in Σ-V, that…".)

14) $A\alpha, \left(F^2M^1xy =+= F^2M^1yx\right) \vdash \left(/\overleftarrow{F^2M^1xy}/ = -/\overleftarrow{F^2M^1yx}/\right)$: by the succession 1 - 12.

Here we are (The formal logical derivation from the assumptions in Σ-V, is finished). Strictly speaking, the formal derivation as such (namely, as *proper formal* one) is nothing but a "skeleton" of the proof. To obtain the proof, it is necessary to interpret the formal inference. F^2M^1xy be interpreted as the evaluation-function "force of action of *y* on movement of x", and let F^2M^1yx be interpreted as the evaluation-function "force of action of x on movement of y". Evaluation-functional meanings of the symbols M^1 and F^2 are precisely defined above (in 2.2) by **Table 1** and **Table 3**, respectively. Using the above-given definitions, one can discover and demonstrate convincingly (by accurate computing relevant compositions of evaluation-tables) that (F^2M^1xy =+= F^2M^1yx): scalar aspect of force of action of *y* on movement of *x* is *formally-axiologically equivalent* to scalar aspect of force of action of *x* on movement of *y*. This wonderful *formal-axiological equation* identifying the scalar evaluation-functions has been considered originally in [42] as a *formal-axiological* "law of contraposition" of the binary operation F^2xy.

However, the content analysis of the physical (mechanical) interpretation gives solid grounds for moving from the *formal-axiological equivalence* of the scalar evaluation-functions to the *formal-axiological equivalence* of the corresponding vectored ones, therefore, in [42], the *formal-axiological law of contraposition* of F^2xy has been vectored. Thus, the *formal-axiological analog* of the third law of Newton's mechanics has been created (discovered or invented) and justified (by computing relevant evaluation-functions) for the first time in [42]. But the discovered (or invented) *analog* is not a statement of "what is"; it is a *formal-axiological* statement of "what is good". Generally speaking, there is a formal-logic gap between "what is" and "what is good". Moreover, there is a habitual faith (intellectually respectable belief) that logical bridging the gap is absolutely impossible.

Although, constructing and demonstrating *the formal-axiological analog* of the classical mechanical law in question has been accomplished and published originally in [42], the mentioned notorious gap has not been logically bridged in [42], and the faith that the gap is logically unbridgeable has not been challenged in [42]. In contrast with and in substantial supplement to [42], for the first time in relevant literature, the present article submits logical bridging the allegedly unbridgeable gap between the law of dynamics and its already existing formal-axiological analog. This logical bridging is the main nontrivial novelty (significant theoretic discovery) of the given article. It is a very important challenge to the dominating paradigm in theoretical physics and in philosophy of science.

The significantly new nontrivial result obtained above in this paper implies that logical bridging the gap is impossible not absolutely but relatively. The faith in its being logically unbridgeable is quite adequate under the *ordinary* condition that knowledge is *empirical*. Nevertheless, under the *extraordinary* condition that knowledge is *a priori*, the gap is logically bridgeable.

According to the above-said, from the two premises, namely, (1) the formal-axiological analog of the third law of Newton's mechanics and (2) the assumption of *a-priori*-ness of knowledge, the third law of Newton's mechanics is formally inferred in Σ-V.

The result of translation of the interpreted formula 11 $\left(\left[\overleftarrow{F^2M^1xy}\right] \leftrightarrow \left[\text{®}\overleftarrow{F^2M^1yx}\right]\right)$ from artificial language of the mathematical model into the natural language of human creatures is the following: "A vectored force of action of *y* on (movement of *x*) exists if and only if the oppositely vectored force of action of *x* on (movement of *y*) exists". The result of translating the interpreted formula 12 $\left(/\overleftarrow{F^2M^1xy}/ = -/\overleftarrow{F^2M^1yx}/\right)$ from the artificial language of mathematical model into the natural language of humans is the following: "an actually existing quantity-magnitude of vectored force of action of *y* on (movement of *x*) is precisely equal to the negative quantity-magnitude of vectored force of action of *x* on movement of *y*".

4. Discussion

4.1. Using a "Mole Hole" for Logical Deriving "Is" from "Must" or "Good", and for Converse Logical Inferring "Must" or "Good" from "Is", in the Formal Axiomatic Epistemology-and-Axiology Theory Σ-V for Obtaining Nontrivial Scientific Results in Mathematical Physics

According to the above-reported results of investigation, within the logically formalized axiomatic epistemology-and-axiology theory Σ-V, if knowledge of nature is *pure a-priori* one, then there is a formal proof, in Σ-V, for the Third Law of Newton's mechanics. Consequently, in the system of *pure a priori* knowledge of nature, modeled by the logically formalized axiomatic theory Σ-V, the famous law of classical dynamics well-known under the name "the Third Newton's law", is strictly grounded by the above-presented formal deductive derivation (from the manifestly indicated and well-defined assumptions).

It is worth highlighting here that Σ-V deviates substantially from the notorious positivism ideal of science philosophy [11]-[18] [29] [45], as Σ-V represents a necessary synthesis of metaphysics (understood as formal axiology) with universal philosophical ontology, and with such universal philosophical epistemology, which recognizes not only empirical but also a *priori* knowledge and combines the two kinds of knowledge consistently. Certainly, synthesizing the mentioned philosophical disciplines while philosophical grounding sciences in general and physics in particular is deviating the positivism on principle. What can one gain from such unhabitual synthesizing? From such paradigm-breaking synthesis the one can gain knowledge of existence of the psychologically unexpected possibility (a hidden "mole hole") for perfectly logical deductive inferring "must" (or "good") from "is", and also for the converse logical deriving "is" from "must" (or "good") in the formal axiomatic epistemology-and-axiology theory Σ-V. Certainly, the wonderful "mole hole" exists (and can be exploited fruitfully) only under some *quite extraordinary concrete condition*, namely, if and only if the knowledge under consideration is *not empirical* but *pure a priori*. Systematical investigating Σ-V (formal proving its theorems) has shown that, in Σ-V, if and only if the knowledge under consideration is *not empirical* but *pure a priori*, then the *statements of existence* ("is"-propositions), the *statements of duty* ("must"-propositions), and the proper *axiological statements* of positive value ("is good"-propositions) exemplified, respectively, by q, Oq, Gq, are logically equivalent and, consequently, mutually substitutable for each other. Hence, for instance, under the indicated *quite extraordinary concrete condition*, substituting "must" for "is" is quite rational in talks and writings of *pure a priori* knowledge. Such abstract theoretical conclusion which appeared in the present article "at the tip of a pen" can be exemplified by the following citation from "Critique of Pure Reason".

Kant writes: "In all communication of motion, action and reaction *must* always be equal" ([31], p. 18). In the cited Kant's proposition, the word "*must*" is italicized by me to highlight the sign of proper *deontic* modality used by Kant instead of the sign of existence. This remarkable linguistic situation is a very important signal of Kant's violating the so-called "Hume Guillotine"—the principle of logical separation (logically unbridgeable gap) between *statements of existence* ("is"-propositions) and corresponding *statements of duty* ("must"-propositions). Certainly, the today dominating formulation of "the Guillotine" has been *ascribed* to D. Hume by his interpreters (opponents and proponents). However, there are some solid grounds for such ascribing in Hume's sceptic doctrine of *empirical* morals, *positive* jurisprudence, and *empirical* knowledge of nature [46] [47] [48].

Kant used to criticize Hume's empiricism systematically. For instance, with respect to philosophy of nature (theoretical physics), he has written the following: "It has hitherto been assumed that our cognition must conform to the objects; but all attempts to ascertain anything about these objects *a priori*, by means of conceptions, and thus to extend the range of our knowledge, have been rendered abortive by this assumption. Let us then make an experiment whether we may not be more successful in metaphysics, if we assume that the objects must conform to our cognition" ([31], p. 7). Another relevant citation from Kant's writings: "Even the main proposition that has been elaborated throughout this entire part, already leads by itself to the proposition: that the highest legislation for nature must lie in our self, *i.e.*, in our understanding, and that we must not seek the universal laws of nature from nature by means of experience, but, conversely, must seek nature, as regards its universal conformity to law, solely in the conditions of the possibility of experience that lie in our sensibility and understanding; …" ([33], p. 71). Finally, it is relevant to take into an account the following Kant's statement: "We must, however, distinguish empirical laws of nature, which always presuppose particular perceptions, from the pure or universal laws of nature, which, without having particular perceptions underlying them, contain merely the conditions for the necessary unification of such perceptions in one experience; with respect to the latter laws, nature and possible experience are one and the same, and since in possible experience the lawfulness rests on the necessary connection of appearances in one experience (without which we would not be able to cognize any object of the sensible world at all), and soon the original laws of the understanding, then, even though it sounds strange at first, it is nonetheless certain, if I say with respect to the universal laws of nature: *the understanding does not draw its* (*a priori*) *laws from nature*, *but prescribes them to it*" ([33], p. 71-72).

Attentively analyzing all the above-provided citations from Kant's writings, it is easy to notice and recognize that while writing of *pure a priori* knowledge of nature, Kant has used the *deontic* modality "must (obligatory, prescribed)" as a synonym of/for the alethic modality "necessary". Was it a contingent blunder by negligence? I believe that no: it was not a contingent mistake but a necessary action accomplished on principle. But, generally speaking, in modal logic, the two kinds of modalities ($O\beta$ and $\Box\beta$) are not logically equivalent [49], consequently, in relation to the above-provided citations, there is a possibility of accusing Kant of committing the blunder in philosophical (modal) logic. However, the odd Kant's using "necessary" and "prescribed" as logically equivalent modalities while his discourse of *pure a priori* knowledge of necessarily universal laws of nature can be successfully explained and completely vindicated in Σ-V by the theorem scheme $\left(A\alpha \supset \left(O\beta \leftrightarrow \Box\beta\right)\right)$, in which the special condition of/for the equivalence is manifestly indicated, namely, the condition ($A\alpha$) of *a-priori*-ness of knowledge. Thus, in perfect accordance with Kant, who has affirmed that the Third Newton Law is an *a-priori* known necessarily universal law of nature prescribed to nature by physicist's understanding, from the *formal-axiological analog* of the Third Newton Law *it follows logically* in Σ-V that a vectored force of action of *y* on (movement of *x*) exists if and only if the oppositely vectored force of action of *x* on (movement of *y*) exists. Certainly, the statement "existence of force of action of *y* on (movement of *x*) is logically equivalent to existence of force of action of *x* on (movement of *y*)" is a statement of what is. But this statement of existence is *logically derived* in the present article from *formal-axiological* and *deontic* premises owing to the wonderful "mole-hole" discovered (or created) in Σ-V. From the pure theoretic viewpoint, it does not matter whether this "mole-hole" is discovered accidentally or created ("dug") intentionally in [37]. What really matters is actual existence of the "mole-hole" which could be used by theorists systematically. In this respect, not only $(A\alpha \supset (O\beta \leftrightarrow \Box\beta))$, but also the wonderful theorem schemes $(A\alpha \supset (G\beta \leftrightarrow O\beta))$, and $(A\alpha \supset (G\beta \leftrightarrow \Box\beta))$, are worth being taken into an account.

4.2. Formal Axiomatic Epistemology Theories Ξ, Σ, Σ+C, Σ-V, and the Controversy between O. Neurath and K. Popper about Philosophy of Science

The hitherto never published substantially new investigation result reported above in the paragraph 3 of this article, is obtained deductively in Σ-V due to some special aspects of some of the nontrivial axiom schemes, namely, AX-12 and AX-13. Up to the present time, the axiom-schemes AX-12 and AX-13 have been never published and discussed elsewhere. However, in Σ-V, there are also some other nontrivial axiom-schemes, namely, AX-3:
$A\alpha \leftrightarrow (K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta)))$ and AX-4:
$E\alpha \leftrightarrow (K\alpha \,\&\, (\Diamond\neg\alpha \vee \Diamond S\alpha \vee \neg\Box(\beta \leftrightarrow \Omega\beta)))$, some special aspects of which are worth discussing. The axiom-schemes AX-3 and AX-4, which belong to (are common for) the logically formalized *axiomatic epistemology theories* Ξ [39], Σ [37], Σ+C [38], and Σ-V (see the above-given definition), are ground-breaking (challenge-making) for the dominating (habitual) paradigm in proper philosophical theory of knowledge.

In the present Section 4.2 of this article, let us discuss the special aspects of these axiom schemes. By the above-given definition, in Σ-V, AX-3, represents an exact criterion (complex one) of *a-priori*-ness of knowledge; AX-4, represents an exactly formulated complex criterion of *empirical*-ness of knowledge. By virtue of the two indicated axiom schemes, the *a-priori* knowledge and the *empirical* knowledge are separated effectively and connected logically within one universal theory of knowledge.

The controversy mentioned in the title of this section of the article has been related to *science* understood as *empirical* cognition of the world as totality of *facts*. According to the old tradition (custom) in epistemology, there are only two criterions of proper *empirical* (in particular, proper *scientific*) knowledge, namely, its *verifiability* and its *falsifiability*. In the axiom-scheme AX-4, devoted to proper *empirical* (proper *scientific*) knowledge, the *verifiability* criterion is represented by the disjunct $\Diamond S\alpha$, and the *falsifiability* criterion is represented by the disjunct $\Diamond\neg\alpha$.

In the axiom-scheme AX-3, devoted to pure *a priori* knowledge (proper rational one), *impossibility of sensual verification* of a knowledge as a *necessary* condition of the knowledge' s being *a priori* is represented by the conjunct $\neg\Diamond S\alpha$, and *impossibility of falsification* of a knowledge as a *necessary* condition of the knowledge' s being *a priori* is represented by the conjunct $\neg\Diamond\neg\alpha$.

Is the conjunction $(\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha)$ a *necessary and sufficient* condition of *a-priori*-ness of knowledge of α? The question only seems trivial (it looks too simple from the viewpoint of the very old epistemic paradigm; but, generally speaking, this question is nontrivial one worthy of pondering over. From the habitual point of view confined within the dominating paradigm in epistemology, the answer to the question is positive; this is an "irrefutable presumption" underlying the paradigm. Since ancient times to our days, possibility of existence of some other (significantly different but quite exact and objective) special criterions of *empirical* (*scientific*) knowledge has been not well-recognized; it has been either missed or ignored. Thus, the set of quite exact objective criterions of *empirical-ness* (*scientifical-ness*) has been reduced to the two: *verifiability* and *falsifiability*. Hence, according to the old but still dominating paradigm, if knowledge is neither verifiable nor falsifiable, then it is not *empirical* (*scientific*) one. Therefore, being subjected to (or guided by) the old tradition (custom) in philosophical epistemology, Kant's discourse (of knowledge in proper mathematics and logic) had arrived necessarily to the following conclusion: any *proper mathematical knowledge is not empirical but pure a priori*, as it is neither falsifiable nor verifiable by sensual experience. (That is why, in particular, according to the dominating semantics of contemporary natural English language, although physics is a science, proper mathematics is not a science.)

Within the two-fold-criterion paradigm in philosophy of science, some thinkers have concentrated mainly on discussing *possibility of verification by sensual experience* [16] [17] [18] [29], etc., and some thinkers have concentrated mainly on discussing *possibility of falsification*. For example, K. Popper [50] [51] is well-known as one of those scientists who have been elaborating and popularizing mainly the falsification-ism emphasizing that scientificalness (empirical-ness) of knowledge implies its falsifiability, hence, impossibility of falsification of knowledge implies its being not a proper *scientific* (actually empirical) knowledge.

Thus, it is a fact of history of philosophy of science that Popper has been intentionally developing *the falsification-ism as an alternative to the verification-ism* [50] [51]. However, it is also a fact of history of philosophy of science that O. Neurath [45] has criticized Popper for being too fixed (excessively concentrated) on *falsifiability* of knowledge as a criterion of its scientificalness. Neurath has insisted that there is a *variety* of qualitatively different forms of *empirical* knowledge, and this variety is not completely reducible to falsifiable knowledge [45]. Of the Popper-Neurath debate on *the falsification-ism as an alternative to the verification-ism* see also [52]. In my opinion, the discrepancy between Popper's and Neurath's philosophies of science is well-modeled by the disjunct $\neg\Box(\beta \leftrightarrow \Omega\beta)$ in the axiom-scheme AX-4. Accepting this axiom-scheme necessitates definitely *negative* answering to the above-formulated nontrivial question of *sufficiency* of the condition $(\Diamond\neg\alpha \vee \Diamond S\alpha)$ for empirical-ness of knowledge, namely, according to AX-4, knowledge can be empirical (scientific) even when it is neither verifiable nor falsifiable one. Certainly, Neurath has not formulated the alternative $\neg\Box(\beta \leftrightarrow \Omega\beta)$ manifestly as it has not been available to him at all. For the first time, the alternative $\neg\Box(\beta \leftrightarrow \Omega\beta)$ has been invented (deliberately constructed), manifestly formulated, and inserted into the relevant axiom-schemes of the multimodal axiomatic epistemology theories Ξ, Σ, Σ+C in [37] [38] [39], respectively. Nevertheless, the opponent of Popper could be considered as a creator of the not quite clear guess (abstract intuitive hypothesis) that, generally speaking, the compound objective criterion of scientificalness (empirical-ness) of knowledge is made up not by the two alternatives exclusively but by many (more than two) qualitatively different ones; reducing the criterion to the pair is not adequate. In the theories Ξ, Σ, Σ+C, Σ-V, Neurath's vague intuition-guess has been clarified, explicated, substantially transformed, and embodied into the precisely formulated disjunct $\neg\Box(\beta \leftrightarrow \Omega\beta)$ which is a very important (heuristically significant) special aspect of the nontrivial axiom-scheme AX-4.

5. Conclusions

From the above-said the following conclusions follow logically. Within the relevant physical interpretation of the formal theory Σ-V, such a *formal-axiological equation* can be constructed and justified by computing relevant evaluation-functions, which equation is a *formal-axiological analog* of the Third Newton's Law of mechanics. Moreover, from the above-presented results, it follows logically that the Third Newton's Law is formally derivable in the above-defined formal axiomatic theory Σ-V (given its relevant physical interpretation) from conjunction of the two nontrivial assumptions precisely formulated and justified in algebra of formal axiology and in the axiomatic epistemology. These nontrivial conclusions are challenging for those who are completely determined by the dominating paradigm grounded on the presumption of absolute impossibility of logical bridging the gap between "is" and "is good" (*i.e.* between ontology and axiology, respectively). According to the above-said, the impossibility is not absolute but relative, as there is a hitherto unknown "mole-hole" in Σ-V for logical bridging the gap in spite of "Hume's Guillotine". The "Guillotine" is quite a universal principle for the realm of *empirical* knowledge exclusively, while the "mole-hole" is located within the domain of *pure a-priori* knowledge exclusively, *i.e.* beyond the realm of *empirical* knowledge.

Taking the above-said into an account, I guess that it is a verisimilar hypothesis that the "mole-hole" created and deliberately used in the present article for logically deriving the Third Newton's Law (in relevantly interpreted Σ-V) from conjunction of 1) the epistemic assumption of *a-priori*-ness of knowledge and 2) the relevant *formal-axiological equation*, can be used again in relation to another strictly universal physical law belonging to the system of *pure a-priori* knowledge of nature. Which necessarily universal physical law is meant in the hypothesis? Well-grounded answering this nontrivial question necessitates future investigations in both physics and applied mathematics.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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