Journal of Applied Mathematics and Physics, 2022, 10, 2459-2498
https://www.scirp.org/journal/jamp
ISSN Online: 2327-4379
ISSN Print: 2327-4352

Formally Inferring Galileo Galilei Principle of Relativity of Motion in an Axiomatic System "Sigma+V" from a Triple of Nontrivial Assumptions

Vladimir Olegovich Lobovikov

Laboratory for Applied System Investigations, Ural Federal University, Yekaterinburg, Russia
Email: vlobovikov@mail.ru

How to cite this paper: Lobovikov, V.O. (2022) Formally Inferring Galileo Galilei Principle of Relativity of Motion in an Axiomatic System "Sigma+V" from a Triple of Nontrivial Assumptions. *Journal of Applied Mathematics and Physics*, **10**, 2459-2498.
https://doi.org/10.4236/jamp.2022.108167

Received: July 6, 2022
Accepted: August 16, 2022
Published: August 19, 2022

Abstract

The present paper submits a result of applying a hitherto unknown logically formalized axiomatic axiology-and-epistemology theory "Sigma+V" to the relativity principle formulated by Galileo Galilei. By this application, the author has continued checking the remarkable (paradigm-breaking) hypothesis that *formal-axiological* interpreting strictly universal laws of classical theoretical mechanics could have a heuristic value for the theory proper. Along with systematical studying proper *algebraic* structure of formal axiology of nature, the *axiomatic* (hypothetic-deductive) method is used in this research as well. The investigation accomplishments are the followings. Galileo Galilei principle of relativity of motion has been represented in a two-valued algebraic system of formal axiology by a wonderful formal-axiological equation which could be called a "*formal-axiological analog* of Galileo relativity principle". A precise definition of that algebraic system is given. The remarkable formal-axiological equation has been created (and checked) in that algebraic system by attentive computing relevant compositions of evaluation-functions. Precise definitions of the relevant evaluation-functions are accomplished by tables. The remarkable formula modeling Galileo Galilei principle of relativity of motion (given the appropriate interpretation of the formal theory) has been formally-logically inferred within Sigma+V from a couple of nontrivial assumptions, namely, 1) a precisely defined assumption of *a-priori*-ness of knowledge, 2) the above-mentioned *formal-axiological analog* of the relativity principle by Galileo Galilei. A not-manifest but quite exact axiomatic definition of "*a-priori*-ness of knowledge" is provided. The formal-logical inference is performed in perfect accordance with the mathematical rigor norms formulated within the formalism doctrine by D. Hilbert, therefore, examining the formal deductive inference submitted in the paper can be accomplished easily. Being a *nontrivial scientific novelty* for proper theoretical physics, hitherto the formal-logical derivation has not been published and discussed elsewhere.

Keywords

Galileo-Galilei-Principle-of-Relativity-of-Motion, Formal-Axiomatic-Theory-Σ+V, Two-Valued-Algebraic-System-of-Metaphysics-as-Formal-Axiology, Formal-Inference-from-Assumptions, Pure-*a-Priori*-Knowledge

1. Introduction

There is too much tendency to make *separate* and *independent bundles* of both the physical and the moral facts of the universe. Whereas, all and everything is naturally related and interconnected.

Augusta Ada King, Countess of Lovelace. In a letter to Andrew Crosse. [1]

The present article is inspired by that ideal of (and program for) axiomatization and logical formalization of mathematical and physical theories, which (ideal) has been created originally and developed substantially by D. Hilbert [2]-[7]. His wonderful ideal (and corresponding program) has been called "*formalism* (in philosophical foundations of mathematics and physics)". In relation to philosophical grounding theoretical physics, the given article presents a significantly new nontrivial result of applying Hilbert's formalism to philosophical foundations (namely, epistemic, formal-logical, and formal-axiological ones) of rational discourse of *a-priori* aspect of classical theoretical mechanics in general and of Galilean principle of relativity of motion especially.

In the present article, many significant sides and aspects of the formalism ideal and method are accepted, and Hilbert's program of axiomatization and logical formalization of theoretical physics is followed. But, along with using the formalism methodology, the given paper rejects the notorious positivism ideal of nonexistence of metaphysics and axiology (which ideal has been very influential and popular one among physicists of Hilbert's time). Herein I imply the concrete positivistic ideal and program targeted at complete extermination of metaphysics and axiology in natural sciences and especially in physics, which (ideal and program) have been formulated and elaborated by the positivist-minded scientists (especially physicists) and philosophers, for instance, by R. Carnap [8] [9] [10] [11] [12], E. Mach [13] [14] [15], O. Neurath [16], H. Reichenbach [17]-[22], B. Russell [23] [24] [25] [26] [27], M. Schlick [28] [29] [30], L. Wittgenstein [31], etc. Thus, in the given paper, I side with Hilbert's epistemic optimism concerning possibility and heuristic fruitfulness of adequate representation (modeling) various content theories (ill-formulated in too fuzzy and ambiguous natural language of "homo sapience") by formal axiomatic theories precisely formulated in an unambiguous artificial language having perfectly determined syntaxis and well-defined semantics. Nevertheless, I disagree with R. Carnap's and other positivists' belief and proclamations that metaphysics and axiology must be excluded from proper theoretic physics completely [8]-[31]. On the contrary, I believe that some exactly defined aspects of proper philosophy (for example, universal formal logic, universal philosophical epistemology, and universal formal axiology) must be presented in artificial languages of well-constructed formal axiomatic theories of physics proper. (By the way, this nonstandard belief is in perfect accordance with the above-placed extraordinary epigraph taken from the letter written by a mathematician and one of the first famous computer programmers Augusta Ada Lovelace, the Countess.)

Let us start with some notes of the historical-philosophical background (natural associations and perquisites) of the perfectly new nontrivial theoretic investigation result submitted in this paper. In times of Galileo Galilei, R. Descartes, B Spinoza, I. Newton, and G.W. Leibniz, proper theoretical physics had been known under the name "natural philosophy", for instance, Newton's well-known treatise on proper physics had been named "Mathematical Principles of Natural Philosophy" [32]. The classical theoretical physics in general, and Newton's writings on mathematical foundations of natural philosophy [32] [33] especially, had made a strong impression on Immanuel Kant and had influenced substantially on his discourse of metaphysics of nature. In some parts of Kant's "Critique of Pure Reason" [34], "Prolegomena…" [35], and other writings [36], one can find his nontrivial references to the classical theoretical physics, namely, to the first and the third Newton's laws of mechanics. In Kant's theory of synthetic *pure a priori knowledge of strictly universal* (*necessarily necessary*) *principles of nature*, the mentioned laws of Newton's mechanics serve as representative concrete instances of the *pure a priori knowledge* of *necessarily universal* principles of proper theoretical physics.

This fact of history of natural philosophy can be demonstrated by the below-given citations from Kant's writings. "The science of *natural philosophy* (physics) contains in itself synthetical judgements *a priori*, as principles. I shall adduce two propositions. For instance, the proposition, 'In all changes of the material world, the quantity of matter remains unchanged'; or that, 'In all communication of motion, action and reaction must always be equal'. In both of these, not only is the necessity, and therefore, their origin *a priori* clear, but also that they are synthetical propositions" ([34], p. 18). Herein, only the third Newton's principle of mechanics is mentioned. But in the following citation, Kant has mentioned both the first and the third Newton's laws of mechanics. "As to the existence of pure natural science, or physics, perhaps many may still express doubts. But we have only to look at the different propositions which are commonly treated of at the commencement of proper (empirical) physical science—those, for example, relating to the permanence of the same quantity of matter, the *vis inertiae*, the equality of action and reaction, etc.—to the soon convinced that they form a science of pure physics (*physica pura*, or *rationalis*), which well deserves to be separately exposed as a special science, in its whole extent, whether that be great or confined" ([34], p. 19).

According to Kant's philosophy of physics [34] [35] [36] partly exposed by the above-given citations, proper theoretical physics (the perfectly *rational natural philosophy*) is nothing but synthetic *pure a-priori* knowledge of nature. From Kant's point of view, this means that the extraordinary knowledge is nothing but a *necessarily necessary condition for* all possible physical facts, therefore, it is *strictly universal for* any possible physical experience. Moreover, according to the set of remarks titled "To Logic and Dialectics" in "New Paralipomena" by A. Schopenhauer ([37], p. 118), a *nontrivial fundamental analogy* between *pure a-priori* knowledge of laws of proper formal logic (herein Schopenhauer has meant the *classical* two-valued formal logic) and *pure a-priori* knowledge of *strictly universal* laws of nature necessarily exists and must be found, understood adequately, and recognized manifestly. In my opinion, this nontrivial remark by Schopenhauer is heuristically important and directly relevant to the purpose of the present article. Concerning possible modernizing attempts of formal logical deriving qualitatively new nontrivial consequences from conjoining Newton's and Kant's significantly reinterpreted conceptions of proper theoretical physics at the level of artificial language of contemporary symbolic logic and philosophy of science, the following theoretically interesting problem can be formulated. If such a logically formalized axiomatic system of proper philosophical disciplines (epistemology, axiology, etc.) which is perfectly adequate to *pure a priori knowledge of nature* is already given (accidentally found or deliberately constructed on purpose—it does not matter), then, is it possible, in this hypothetical formal axiomatic system, to make (invent) *formal logical* (*deductive*) *derivations* of *pure-a-priori principles of perfectly rational physics* from the nontrivial presumption of *a-priori*-ness *of* knowledge? In my opinion, for progressive development of proper theoretical physics, the indicated nontrivial problem is very important and, obviously, very difficult one.

Today, some significant aspects (subproblems) of the indicated difficult compound theoretical problem are already solved by means of a formal axiomatic theory Σ (originally formulated and published in [38]) and by its various modifications, for instance, Σ+C [39]. In particular, the *formal logical* (*deductive*) *derivation* of the formula representing the *principle of inertia* in the formal axiomatic theory Σ (given that an appropriate physical interpretation of Σ is provided) from the nontrivial assumption of knowledge *a-priori*-ness is already discovered (or invented, *i.e.* constructed on purpose) and published in [40].

The *general* philosophical *motivation* of my making (constructing) formal axiomatic theories for philosophical grounding mathematics and theoretical physics coincides with the general philosophical *motivation* of the above-mentioned D.

Hilbert's *formalism* in philosophical grounding mathematics and theoretical physics [2]-[7]. My *particular* (peculiar, singular) philosophical *motivation* for systematical constructing (inventing) various logically formalized *multimodal* axiomatic *epistemology-and-axiology* systems is based on my tendency to the worldview ideal of a *consistent synthesis* (*fundamental interconnection and unity*) of proper philosophical disciplines (ontology, epistemology, axiology, etc.). According to the ideal, mathematics and physics have not only formal ontological and formal epistemological, but also *formal axiological* grounds. This synthesis ideal is paradigm-breaking (not habitual) one, hence, there is a special motivation (need) for exposing its heuristic potential on concrete examples. Being motivated in this way, I have created the formal *multimodal* axiomatic theory Σ, which is a *consistent synthesis of epistemic and axiological modalities* [38]. Being inspired by the above-cited Lovelace's letter [1], I have discovered a "mole-hole" in Σ, for formal logical deriving deductively "is" from "ought" and conversely [38] [40]. Then, being motivated by the above-cited Lovelace's letter [1], I have decided to attempt to use the "mole-hole" found in Σ, for formal logical deriving deductively some strictly universal laws of theoretical physics in Σ, from very small sets of either-epistemic-or-axiological assumptions well-defined in Σ [40] [41] [42]. One of successful attempts of such using the "mole-hole" in Σ is presented in the article "Formal Inferring the Law of Conservation of Energy from Assuming A-Priori-ness of Knowledge" [41]. Then, being inspired by feeling that the result is positive, I had decided to continue exploiting the "mole-hole" in Σ with respect to some other necessarily universal laws of theoretical physics, for example, in relation to the third Newton's law of mechanics, but in this concrete relation results were negative. Then I had recognized that being well-done for dealing with *scalar* magnitudes, Σ had been not well-done for dealing with *vector* magnitudes, hence, it was necessary to improve Σ substantially by making some progressive developments in it. For the sake of successful realizing the purpose concerning the third Newton's law, I had made a significant mutation in the axiomatic theory Σ by some important additions to it. The undertaken nontrivial improvements, in particular, addition of *vector* symbols to the alphabet of object-language of Σ, and addition of new axiom-schemes concerning *vectors* had resulted in transformation of Σ into a qualitatively new logically formalized axiomatic theory called "Σ-V" which had been significantly richer than Σ. Owing to the substantial improvements, finally, the hard problem had been solved and the intellectual adventure had arrived to the positive result presented in the article "Formally Deriving the Third Newton's Law from a Pair of Nontrivial Assumptions in a Formal Axiomatic Theory Sigma-V" [42].

Writing of *a-priori*-known *strictly universal* principles of proper theoretical physics, Kant used to mention the first and the third Newton's laws as concrete examples of such principles [34] [35]. Today, the two Newton's laws evaluated by Kant as representative examples of *pure-a-priori-known* laws of nature are already derived deductively from small sets of relevant assumptions in either Σ or Σ-V. Is the investigation completely finished? Is there a possibility to move further in the direction indicated somehow by G.W. Leibniz, I. Kant, D. Gilbert, and A.A. Lovelace? The questions are theoretically interesting and nontrivial. In relation to them, it is worth formulating and discussing the following verisimilar hypothesis. Probably, the investigation is not completely finished as the set of *a-priori*-known strictly universal principles of rational philosophy of nature is not exhausted yet. It is worth pondering over the hypothesis that, probably, the set is open (potentially infinite) one.

In this concrete relation and situation, today, the following still not answered theoretically interesting nontrivial question has been raised quite naturally. Can the *principle of relativity of motion* originally quite manifestly recognized and precisely formulated by Galileo Galilei [14] [22] [43] [44] [45] be *formally-logically* (*deductively*) *inferred* in Σ-V (or in a result of its mutation), from conjunction of 1) the assumption of *a-priori*-ness of knowledge and 2) a hypothetical *formal-axiological* equation modeling the relativity principle in the two-valued algebraic system of metaphysics as formal axiology [46]? If a definitely positive answer to this challenging question is to be given, then 1) the assumption of *a-priori*-ness of knowledge is to be precisely defined, 2) the hypothetical *formal-axiological* equation is to be precisely formulated and justified by accurate computing relevant compositions of evaluation-functions in the algebraic system of formal axiology, 3) an easily checkable actually formal logical inference from the two premises is to be invented (constructed) either in Σ-V, or in a result of its mutation. In this article, not exactly the theory Σ-V as such, but a result of its mutation (clarification and rectification) is used. The result of amending is called Σ+V. A precise definition of Σ+V is to be given in section 2.2. of the article.

Thus, the *principal purpose of the investigation* presented in this paper, is making (discovering or intentional constructing) a formal-logical (deductive) derivation (in Σ+V) of such an extraordinary formula which models (under an appropriate physical interpretation of Σ+V) Galileo Galilei principle of relativity of motion, if the assumption of knowledge *a-priori*-ness is accepted. However, for successful realization of the precisely and manifestly formulated *principal purpose of the investigation*, it is indispensable to have sufficiently adequate means, in particular, an appropriate system of basic theoretical concepts and effective mathematical methods.

2. Materials and Methods

2.1. The Method of Mathematical Modeling: An Introduction of a Proper Algebraic Structure Defined on a Set of Abstract Evaluative Forms of/for Concrete Materials (Two-Valued Algebraic System of Formal Axiology)

Materials of the given article belong to theoretical physics and especially to that aspect of it which deals with pure *a priori* knowledge of strictly universal mathematical principles of natural philosophy. For analyzing and organizing the materials, the method of mathematical modeling is used. In this section of the article, I am to introduce the two-valued *algebraic system* of metaphysics as formal axiology, which (algebraic system) is to be exploited as a necessary means for obtaining the main new nontrivial result. Now let us begin with giving precise definitions of basic notions.

The two-valued algebraic system of metaphysics as formal axiology is nothing but a triple <Θ, Ω, R>, in which the sign Θ denotes the set of all such and only such *either-existing-or-not-existing units* which are *either good or bad* ones from the viewpoint of an *evaluator* $\mathcal{E}$. The sign $\mathcal{E}$ denotes a person (individual or collective, natural or artificial one—it does not matter), in respect to which all assessments are performed. Certainly, $\mathcal{E}$ is a *variable*: changing values of $\mathcal{E}$ can result in changing assessments of concrete elements of Θ. However, if a value of the variable $\mathcal{E}$ is fixed, then assessments of concrete elements of Θ are quite definite. Elements of Θ are called formal-axiological-objects of metaphysics. The signs "g" (good), and "b" (bad) stand for *moral values* (*abstract axiological ones*) of elements of Θ. Moral *actions or persons* (individual or collective, natural or artificial ones—it does not matter) are *concrete instances* (*particular cases*) of elements of Θ. In <Θ, Ω, R>, the sign Ω denotes the set of all *n-ary algebraic operations* defined on the set Θ. (These algebraic operations are called *formal-axiological* ones.) In the mentioned triple, the symbol R denotes the set of all *n-ary formal-axiological relations* defined on the set Θ. (For instance, the below-defined binary relation "*formal-axiological equivalence*" belongs to R.)

Algebraic operations, defined on the set Θ, are *value-functions*. *Value-variables* of these functions take their values from the set {g (good), b (bad)}. Here the signs "g" and "b" denote the values "good" and "bad", respectively. The value-functions take their values from the same set.

In the talk of *value-functions*, the following mappings are meant: $\{g, b\} \to \{g, b\}$, if one talks of the functions determined by *one value-argument*; $\{g, b\} \times \{g, b\} \to \{g, b\}$, if one talks of the functions determined by *two value-arguments* (here "×" denotes the Cartesian product of sets); $\{g, b\}^N \to \{g, b\}$, if one talks of the functions determined by *N value-arguments*, (here *N* is a finite positive integer).

In algebra of formal axiology, the signs "*x*" and "*y*" denote *abstract-value-forms* of elements of Θ. (Moral-value-forms of actions and of individual or collective persons are *concrete instances* or particular cases of *abstract-value-forms* of elements of Θ.) Elementary abstract-value-forms deprived of their specific contents represent independent abstract-value-arguments. Complex abstract-value-forms deprived of their specific contents represent abstract-value-functions determined by these arguments. In the present paper, due to its quite definite subject-matter and page limit, only some concrete examples of the functions determined by only *one* value-argument, namely, those which are relevant to the subject-matter and to the research purpose are considered. Let such functions be introduced by the following glossaries for **Table 1** and **Table 2**. It is necessary to take into an account that hereafter in this article, the upper index 1 standing

Table 1. Defining the one-placed value-functions.

x	$R_1{}^1x$	$R_2{}^1x$	$L_1{}^1x$	$L_2{}^1x$	M^1x	$M_1{}^1x$	$D_1{}^1x$	$D_2{}^1x$	$I_1{}^1x$	$I_2{}^1x$	$O_1{}^1x$	$E_1{}^1x$
g	b	g	b	g	b	g	b	g	b	g	b	g
b	g	b	g	b	g	b	g	b	g	b	g	b

Table 2. The value-functions determined by one value-variable.

x	$B_1{}^1x$	$N_1{}^1x$	$B_2{}^1x$	$B_3{}^1x$	$B_4{}^1x$	$M_2{}^1x$	Q^1x	$S_1{}^1x$	$D_3{}^1x$	$D_4{}^1x$	$M_3{}^1x$	W^1x
g	g	b	g	g	g	g	g	b	g	b	b	g
b	b	g	b	b	g	b	b	g	b	g	g	b

immediately after a capital letter means that the indexed letter denotes a one-placed valuation-function. The capital letters may possess no lower number indexes, for example, M^1x in Table 1, and Q^1x in Table 2. If a value-functional symbol (capital Latin letter) does not have a lower number index, and *the same* letter has a lower number index, then the letter possessing lower index is different from *the same* letter without lower index, for example, M^1x and $M_1{}^1x$ (in Table 1) are different signs. A difference of lower number-indexes means difference of the corresponding signs, for instance, in Table 1, $R_1{}^1x$ and $R_2{}^1x$ are different signs.

Glossary for the above-located Table 1. The symbol $R_1{}^1x$ stands for the value-function "*relativity* (or *relativeness*) of (what whom) *x*", or "*x*'s *being in relation*". The symbol $R_2{}^1x$ stands for the value-function "*being in relation to* (what whom) *x*", or "*relativity to x*". The sign $L_1{}^1x$ denotes the value-function "linkage (connection), nexus, bonds of *x*", or "*x*'s being linked, bound, fixed, uneasy". The sign $L_2{}^1x$ denotes the value-function "being linked, bound, fixed, tied with (what whom) *x*", or "linking (connecting), concatenating by (what whom) *x*". The symbol M^1x stands for the value-function "movement (change) of (what whom) *x*". The symbol $M_1{}^1x$—"movement (change) by (what whom) *x*", or "being moved (changed) by *x*". $D_1{}^1x$—"*x*'s being dependent, determined, defined". $D_2{}^1x$—"being dependent, determined, defined by (what whom) *x*". $I_1{}^1x$—"being independent of (what whom) *x*, or "being undetermined, undefined by *x*". $I_2{}^1x$—"*x*'s being independent (free), undetermined, undefined". $O_1{}^1x$—"*x*'s *opposite*", *or* "*opposite of/for* (what whom) *x*". $E_1{}^1x$—"essence (nature) *of x*". The above-mentioned valuation-functions are defined precisely by the above-located Table 1.

Glossary for the above-located Table 2. The symbol $B_1{}^1x$ stands for the value-function "*x*'s *being* (existence)". $N_1{}^1x$—"*x*'s *nonbeing* (nonexistence)". $B_2{}^1x$—"*x*'s *being* by itself". $B_3{}^1x$—"*x*'s *being* by its nature (essence)". $B_4{}^1x$—"*x*'s *being* in itself". $M_2{}^1x$—"*magnitude of quantity of x*", or "*x*'s *quantity magnitude* (scalar one)". Q^1x—"*quickness*, *rapidity* (*speed*) *of x*". $S_1{}^1x$—"*x*'s *slowness*". $D_3{}^1x$—"*x*'s own (inner, free, proper) direction", or "*direction by x*", or "(proper) *vector of x*". $D_4{}^1x$—"external coercive directing (what, whom) *x as an object* (of coercive directing)". M_3^1x—"*matter* (*material*) *of x*", or "*materialness* of (what, whom) x", or "x's being a material (matter)". W^1x—"*world of* (what, whom) x". These valuation-functions are defined precisely by the above-located **Table 2**.

For precise tabular definitions, content discussions, and algorithmic computations of compound compositions of many other *value-functions* determined by *one* value-argument, see [38] [39] [40] [41] [42] [47] [48] [49] [50] [51].

Now let us leave the one-placed value-functions for two-placed ones. The binary functions are introduced by the following glossary. It is worth reminding and taking into an account that hereafter in this article, the upper index 2 standing immediately after a capital letter means that the indexed letter denotes a *two-placed* valuation-function. Also, it is worth reminding here that a difference of lower number-indexes means difference of the corresponding signs. For instance, in the following glossary, I_1^2xy, I_2^2xy, I_3^2xy, I_4^2xy, I_5^2xy, and I_6^2xy are six different signs. The symbols standing for value-functions may have no lower number-indexes, for instance, M^2xy, R^2xy, L^2xy, M^1x, W^1x. However, when a sign does not possess a lower number-index, then that sign is different from the same sign possessing a lower number-index. For instance, M^1x and M_1^1x are different signs.

Glossary for the below-located **Table 3** and for the *analytical* definitions Def1-Def7. The symbol M^2xy stands for the value-function "movement (change) of x by (what whom) y", or "x's being moved (changed) by y". The sign R^2xy denotes the value-function "x relative to y", or "x in relation to y", or "relativity of x to y", or "x's being relative to y". The symbol L^2xy stands for the value-function "linkage (connection, bounds) of x with y", or "x's being linked, bound, fixed to/with y". The sign S^2xy denotes the value-function "x's subjection to (dependence from) y", or "x's being dependent (subject) of y". The symbol I_1^2xy—"interconnection of x and y". The sign I_2^2xy—"interrelation of x and y". D^2xy—"x's being determined (defined) by y". I_3^2xy—"inter-determination (inter-definition) of x and y". A^2xy—"y's *action on* x". I_4^2xy—"interaction of x and y". K^2xy—"joint existence of x and y", or "being of both x and y together", or "*uniting* (conjoining) x and y (in a whole)". G^2xy—"*genesis* (*creation*) *of y from* x". O^2xy—"y's opposition to (or contradiction with) x". I_5^2xy—"inter-opposition (or inter-contradiction) of x and y". I_6^2xy—"inter-subjection (or inter-dependence) of x and y". C^2xy—"existence of y in x". E^2xy—"equalizing (what, whom) x and y", or "treating x and y as possessing identical values". B^2xy—"x *by* (what, whom) y", or "*x's being by y*".

Table 3. Defining the two-placed value-functions.

x	y	M^2xy	R^2xy	L^2xy	S^2xy	D^2xy	A^2xy	K^2xy	G^2xy	O^2xy	C^2xy	B^2xy
g	g	b	b	b	b	b	b	g	b	b	g	g
g	b	b	b	b	b	b	b	b	b	b	b	g
b	g	g	g	g	g	g	g	b	g	g	g	g
b	b	b	b	b	b	b	b	b	b	b	g	b

Only some of the mentioned two-placed value-functions are precisely defined above by Table 3. Some of them are precisely defined *not tabularly but analytically* by means of the *formal-axiological equivalences* Def1-Def7 placed immediately after Table 3.

AnDef1: I_1^2xy =+= $K^2L^2xyL^2yx$.

AnDef2: I_2^2xy =+= $K^2R^2xyR^2yx$.

AnDef3: I_3^2xy =+= $K^2D^2xyD^2yx$.

AnDef4: I_4^2xy =+= $K^2A^2xyA^2yx$.

AnDef5: I_5^2xy =+= $K^2O^2xyO^2yx$.

AnDef6: I_6^2xy =+= $K^2S^2xyS^2yx$.

AnDef7: E^2xy =+= $K^2C^2xyC^2yx$.

For understanding the analytical definitions AnDef1-AnDef7, it is necessary to have an exact definition of the meaning of "=+=", therefore, let us immediately formulate it precisely.

Definition DEF-1. In the two-valued algebraic system of formal axiology, any value-functions Ξ and Θ are *formally-axiologically equivalent* (this is represented by the expression "Ξ =+= Θ"), if and only if they acquire identical values (from the set {g (good), b (bad)}) under any possible combination of the values of their value-variables.

Definition DEF-2. In the two-valued algebraic system of formal axiology, any valuation-function Θ is called *formally-axiologically* (*or necessarily*, *or universally*, *or absolutely*) *good* one, or a *law of algebra of formal axiology* (or a "law of algebra of metaphysics"), if and only if Θ acquires the value g (good) under any possible combination of the values of its value-variables. In other words, the function Θ is *formally-axiologically* (*or constantly*) *good* one, iff Θ =+= g (good).

Definition DEF-3. In the two-valued algebraic system of formal axiology, any valuation-function Θ is called *formally-axiologically* (*or invariantly*, *or absolutely*) *bad* one, or a "*formal-axiological contradiction*", if and only if Θ acquires the value *b* (*bad*) under any possible combination of the values of its value-variables. In other words, the function Θ is *formally-axiologically* (*or necessarily*, *or universally*, *or absolutely*) *bad* one, iff Θ =+= b (bad).

Concerning the two-placed relation "=+=", the following very important linguistic fact must be taken into an account. At the level of natural human language, which is essentially ambiguous and vague one, as a rule, the binary relation "Ξ =+= Θ" is represented by the words and word combinations "is", "is equivalent to", "means", "implies", "entails". In the natural language of human beings, these words and word combinations are homonyms and homophones: they and their utterances (pronouncements) may denote the *formal-axiological equivalence* relation "=+=". As, in the not quite clear natural language, along with having the above-defined *formal-axiological* meanings, the homophones "is", "means", "implies", "is equivalent to" may have also the formal-logic meanings (herein I imply the logic operations "equivalence" and "implication"), there is a real possibility of logic-linguistic confusions generated by complete identifying and, consequently, substituting for each other the qualitatively different concepts "=+=" and "↔" (logic operation "equivalence"), or "=+=" and "⊃" (logic operation "implication"). Such chaotic mixing and substituting are strictly forbidden in the two-valued algebraic system of formal axiology: violating this ban can generate shocking illusions of grave paradoxes.

Using the above-given exact definitions of notions and functions, any intellectual system (natural or artificial—it does not matter) can produce the following finite (but potentially infinite) succession of formal-axiological equations. The readers are invited to examine all the below-located equations themselves to be confident that they are really valid. To help readers to understand the equations, to the right after each equation immediately after the colon, I place a translation from the artificial language into the natural human one.

1) $B_1^1x =+= M^1M_3^1x$: being of x is *formally-axiologically equivalent* to movement of matter of x.

2) $N_1^1M^1M_3^1x =+= N_1^1x$: nonbeing of movement of matter of x is *formally-axiologically equivalent* to nonbeing of x.

Checking the formal-axiological equations 1) and 2) is quite elementary. However, although, in first approximation, the equations are not in a conflict with ordinary (statistically normal) intuition of physicist, the reader may be disappointed by feeling that they are somewhat trivial (too simple) ones. Certainly, the technical simplicity and content triviality of 1) and 2) are quite expectable results of limiting the discourse domain to *one-placed* functions *exclusively*. Therefore, responding to that somewhat critical hypothetic feeling, let us move immediately to some not so simple equations representing some more fine and profound ideas. Certainly, reducing the technical simplicity and content triviality is to be a quite expectable result of adding *two-placed* functions to the discourse domain. Thus, not only *one-placed* functions but also *two-placed* ones are to be used hereafter for construction of compositions of functions under investigation.

3) $R^2N_1^1xy =+= R^2N_1^1yx$: (nonbeing of x) in relation to y is *formally-axiologically equivalent* to (nonbeing of y) in relation to x. This is a hitherto not recognized nontrivial *principle of relativity of nonbeing*, which is very interesting for proper theoretical philosophy and necessary for systematical mathematizing (modeling) proper theoretical knowledge of nature.

4) $R^2M^1xy =+= R^2M^1yx$: (x's movement) in relation to y is *formally-axiologically equivalent* to (y's movement) in relation to x. One can be strongly surprised by recognizing an amazing *similarity* between the equations 3) and 4). In my opinion, from the viewpoint of proper theoretical philosophy and proper theoretical physics, the enigmatic *similarity* is very important. Also, one can feel a strongly surprising *similarity* between the equation 4) and the famous principle of relativity of motion by Galileo Galilei.

5) $R^2Q^1M^1xy =+= R^2Q^1M^1yx$: quickness (speed, rapidity) of x's movement in

relation to y is *formally-axiologically equivalent* to quickness (speed, rapidity) of y's movement in relation to x. Herein, the astonishing *similarity* with Galileo's principle of motion relativity becomes significantly stronger. It makes up an odd but heuristically important puzzle. What does this odd puzzle allude to (hint on)? I guess that this hypothetical allusion is such a substantially innovative nontrivial problem which is worth taking seriously and investigating systematically.

Here, it is worth taking into an account also a wonderful *similarity* with the well-known *formal-logic law of contraposition*. Usually (as a *statistical* norm), in academic literature on mathematical logic, the word combination "*law of contraposition*" means the law of *contraposition of classical* ("*material*" *or Philonian*) *implication*. (Originally, the *truth-functional* meaning of the conditional had been recognized and defined precisely by Philo the logician of Magara.) Let such a binary operation of two-valued algebra of logic which (operation) is *mathematically dual* to the classical (material) implication, be called "correction". Usually, in natural language, the logic operation "correction" is represented by "not-A, but B", where A and B are sentences. In the artificial language of symbolic logic, the binary operation called "correction" is represented by the formula $((\neg A)$ & $B)$, where & means the classical *conjunction* and $(\neg A)$ means the classical *negation* of the sentence A. In algebra of classical mathematical logic, it is not difficult to find out and justify deductively an almost unknown *formal-logic law of contraposition of the correction*. The recognition and justification can be accomplished either by computing truth-tables, or by applying the well-known *logic principle of duality* to the logic law of contraposition of classical (material) implication. (In algebra of logic, the principle of duality affirms that if two logic functions are logically equivalent, then such logic functions which are *mathematically dual* to them, are logically equivalent ones as well.) Thus, according to the logic principle of duality, if the contraposition of the implication is valid then the contraposition of the correction is valid too. This abstract talk of the two qualitatively *different* contraposition laws (in mathematical logic), is quite relevant to the subject-matter of the present article, because by means of the above-given definitions, one can discover that the tables defining *formal-axiological* meanings of formulae R^2M^1xy and R^2M^1yx are *formal-axiological analogues* of the tables defining truth-functional meanings of the binary logic operations called "correction". According to the above-said, let either of the equations 3), 4), and 5) be called hereafter "a *formal-axiological law of contraposition* of the binary operation R^2xy (*x relative to y*)".

Originally, the psychologically unexpected (surprising) *formal-axiological equivalences* 4) and 5) representing the *formal-axiological law of contraposition* of the binary operation R^2xy (*x relative to y*), have been recognized manifestly, formulated precisely, and justified convincingly by computing compositions of relevant value-functions, in the article [46]. Nevertheless, in the present article, it is indispensable to understand perfectly and to emphasize especially, that the wonderful equations 4) and 5) of the indicated algebraic system, which (equations) have been discovered and discussed initially in [46], are exactly *formal-axiological* (ones of that algebraic system). This means that they are not statements of "what is" but statements of "what is good". Hence, strictly speaking, the algebraic justification of 4) and 5) by computing relevant evaluation-tables is not a proper formal logical proof of Galileo's relativity principle, which (principle) is a statement of "what is". Certainly, the remarkable equations 4) and 5) do make up a *formal-axiological analog* of Galileo relativity principle, but strictly speaking, in fact, the algebraic justification of the *analog* is not quite sufficient for actually formal logic grounding Galileo's principle of mechanics.

Thus, still the motion-relativity principle of proper theoretical physics affirming of *what is* (in nature), is grounded by formally-logically (deductively) deriving it (from small number of acceptable axioms and assumptions) neither in [46] nor in any other publication. A realization of the hitherto not fulfilled goal of formal logical (deductive) deriving Galileo relativity principle (of *what is* in nature) from a couple of relevant nontrivial assumptions (in a logically formalized axiomatic theory) is the main purpose of the present article. The nontrivial result of realizing this purpose has not been published elsewhere. One of the mentioned nontrivial assumptions, namely, the *formal-axiological analog* of Galileo's relativity principle is already formulated precisely and substantiated above in the present section of this article (by computing compositions of relevant value-functions in the two-valued algebraic system of formal axiology (see the above-located equations 4 and 5). The second of the mentioned nontrivial assumptions is the epistemological presumption of knowledge *a-priori*-ness. Therefore, successfully to fulfill the above-formulated purpose, it is necessary to have quite an exact definition of the epistemic notion "*a priori* knowledge about (what, whom) *q*". A sufficiently precise definition of this important notion of epistemology is given below in the next section of this article. The definition is given by means of a formal axiomatic theory. Hence, it is such an indirect (axiomatic) definition which is in accordance with Hilbert's formalism ideal. Although it is not a manifest definition, it is sufficiently precise and acceptable one for intentional constructing and scrutinizing the hypothetical formal inference. Now let us move directly to precise axiomatic defining the nonstandard (unhabitual) epistemic modality "agent *a-priori* knows that *q*".

2.2. The Axiomatic Method at Work: Such an Unknown Logically Formalized Axiomatic System Called "Sigma+V" Which Has Been Invented Intentionally for Deductive Organizing a Hypothetical System of *a-Priori* Known Necessarily Universal Laws of Theoretical Mechanics

Concerning possibilities of fruitful using axiomatic method in theoretical physics, it is worth making acquaintance with [6] [20] [52] [53] [54]. Especially, interesting attempts to axiomatize some concrete aspects of theoretical physics have been undertaken by D. Hilbert [6] [52] and H. Reichenbach [20]. Thus, the precedent has been made. Now, let us apply it to the substantially analogous case.

As the main purpose of the present paper is formal axiomatic grounding the relativity principle manifestly formulated and systematically advocated by Galileo Galilei, it is indispensable to provide a precise definition of that formal axiomatic theory, which is to be used for realizing the purpose. Moreover, as Galileo's principle of relativity of motion is to be derived deductively within some logically formalized axiomatic theory (given its relevant physical interpretation) from that pair of assumptions which contains the assumption of *a-priori*-ness of knowledge, also, it is indispensable to provide a precise definition of "*a-priori*-ness of knowledge". These two sufficiently exact definitions are to be given below in this section of the article, by virtue of an inevitably complicated but perfectly strict definition of the unknown formal axiomatic theory called "Σ+V", which is a result of mutation (rectification) of the almost unknown theory Σ-V published only in [42].

The formal axiomatic theory Σ+V is an outcome of significant mutations in (and additions to) the logically formalized axiomatic *epistemology-and-axiology* systems Σ [38] [40] [41], Σ+C [39], and [42]. For constructing a sufficiently precise definition of the formal axiomatic system Σ+V, it is indispensable to provide exact definitions of its basic concepts, namely, the concept "*alphabet* of object-language of Σ+V", the abstract syntactic notion "*term* of Σ+V", the abstract syntactic concept "*formula* of Σ+V", and, finally, the fundamental notion "*axiom* of Σ+V". Exact definitions of the mentioned basic concepts of Σ+V are *analogous* to the definitions of corresponding concepts of Σ [38] [40] [41] and Σ+C [39]. However, strictly speaking, in the given paper, it is necessary to provide exact definitions of the concept "*alphabet* of object-language of Σ+V", the notion "*term* of Σ+V", the general concept "*formula* of Σ+V", and the basic notion "*axiom* of Σ+V", notwithstanding the mentioned *analogousness*, because, strictly speaking, "*analogy*" *is not logically equivalent to* "*identity*". The above-mentioned basic concepts (of Σ and Σ+C) are not in relation of proper identity (which is an equivalence one) to the corresponding *analogous* concepts of Σ+V. The proper identity relation is transitive, while, generally speaking, the *analogy* relation is not transitive one. Consequently, although the impression (sensation) of repetitions of the already published statements is truthlike, in fact, rigorously speaking, it is false (illusory). Therefore, now I am to begin formulating the rigorous definitions necessary for adequate understanding the present paper.

First of all, it is necessary to fix the meaning of "alphabet of object-language of Σ+V". According to the definition, the alphabet of object-language of Σ+V includes all the symbols which are contained in the alphabet of object-language of the logically formalized theory Σ. However, the conversion of this prpposition is not true, because, in Σ+V, some novel signs are added to both: the alphabet of Σ, and the alphabet of Σ+C. The result of such substantial changes (complements) is the below-located rigorous definition of the notion "alphabet of object-language of Σ+V".

1) The lowercase Latin letters p, q, d (and these letters having lower number indexes) belong to the alphabet of object-language of Σ+V; these lowercase Latin letters are named "*propositional* (or *sentential*) letters". In the alphabet of object-language of Σ+V, *not all lowercase Latin letters are called propositional ones* because, according to the given definition, those lowercase Latin letters which belong to the set {g, b, e, n, x, y, z, t, f} do not belong to the set of *propositional* letters of object-language of Σ+V.

2) The habitual logic symbols ¬, ⊃, ↔, &, ∨ named, respectively, "classical negation", "classical (or 'material') implication", "classical equivalence", "classical conjunction", "classical not-excluding disjunction" belong to the alphabet of object-language of Σ+V.

3) Elements of the set {□, K, A, E, S, T, F, P, D, C, G, W, O, B, U, J, E} are elements of the alphabet of object-language of Σ+V as well. They are named "modality symbols" in Σ+V.

4) The signs "→" and "←", called "vector symbols" or "arrows" ("left-right arrow" and "right-left one") belong to the alphabet of object-language of Σ+V. In comparison with the formal theories Σ and Σ+C, the vector symbols are novelties as the "arrows" do not belong to the alphabets of object-languages of the already published and investigated formal theories Σ and Σ+C. Moreover, with respect to proper *physical* interpretations of formal theory Σ+V, the vector symbols belonging to its alphabet are very important for possible fruitful innovations and applications of that theory. By the way, the vector symbols are not habitual (and even very unusual, odd) for object-languages of formal theories based on classical symbolic logic. Thus, even at the level of its alphabet, Σ+V differs much from the theories Σ and Σ+C.

5) The lowercase Latin letters *x*, *y*, *z* (and also these letters having lower number indexes) belong to the alphabet of object-language of Σ+V. Such and only such letters are named "*axiological variables*" in Σ+V.

6) The lowercase Latin letters "g" and "b" named "*axiological constants*" also are elements of the alphabet of object-language of Σ+V.

7) The capital Latin letters having number indexes—E^1, C^1, K^1, K^2, E^2, C^2, C_j^n, B_i^n, D_m^n,A_k^n, … are elements of the alphabet of object-language of Σ+V (such capital Latin letters are named "*axiological-value-functional symbols*"). Here the upper number index *n* informs that the indexed axiological-value-functional symbol is *n*-placed one. Absence of the upper number index indicates that the value-functional symbol is determined by only one axiological variable. The axiological-value-functional symbols may possess no lower number index. If a value-functional symbol does not have a lower number index, and the same symbol has a lower number index, then the symbol with lower index is different from the same symbol without lower index, for example, M^1x and M_1^1x are different signs. If value-functional symbols possess lower number indexes, then, if these

indexes are different, then the indexed functional symbols are different ones, for example, I_3^2xy and I_4^2xy are different signs.

8) The signs "(", ")" named "round brackets" are elements of the alphabet of object-language of Σ+V as well. These auxiliary signs are utilized in the present article as usually in symbolic logic, namely, as pure technical symbols.

9) The signs "[", "]" ("square brackets") are elements of the alphabet of object-language of Σ+V also. However, it is worth emphasizing here that in contrast to the "round brackets", in Σ+V, the "square brackets" are used not as the habitual pure technical symbols, but as *ontologically meaningful* signs. Such non-standard using the "square brackets" is psychologically unexpected (unhabitual) one. In relation to natural language psychology, square brackets and round ones seem identical as very often in natural language they are used as synonyms. But in the object language of Σ+V, the two kinds of brackets possess *significantly different* meanings (play substantially different roles): usage of round brackets is purely technical (auxiliary) one, while square-bracketing possesses an *ontological* meaning. The *ontological* meaning of square-bracketing is defined below in that part of the present paper which is devoted to *semantics* of object-language of Σ+V. Nevertheless, even at the level of syntaxis of the artificial object language of Σ+V, square brackets *play a substantial role* in the precise definition of the concept "formula of Σ+V". (This definition is to be given below in this section of the article.) Moreover, square-bracketing *plays a substantial role* in the precise formulations of some axiom-schemes of Σ+V" (which formulation are to be given below also in this section of the article).

10) An unhabitual artificial symbol "=+=" named "*formal-axiological equivalence*" is an element of the alphabet of object-language of Σ+V. The odd symbol "=+=" *plays a substantial role* in the precise definition of the concept "formula of Σ+V" and also in the precise formulations of some axiom-schemes of Σ+V.

11) The habitual symbols "-" (negative-number sign called "minus") and "=" (equality of numbers) from the language of arithmetic are elements of the alphabet of object-language of Σ+V.

12) The habitual symbol "/" also belongs to the alphabet of object-language of Σ+V, although, in Σ+V, this quite habitual symbol is used in a quite unexpected (unhabitual) special meaning (to be defined precisely below while formulating semantics of Σ+V).

13) The habitual symbol ® belongs to the alphabet of object-language of Σ+V, but quite an unhabitual role (meaning) of ® in Σ+V differs much from the well-known habitual one.

14) A sign is an element of the alphabet of object-language of Σ+V, if and only if the sign belongs to this alphabet due to the above-formulated items 1)-13) of the given definition.

Any finite chain (queue) of symbols is named "an *expression* of the object-language of Σ+V", then and only then, when that chain contains such and only such signs which are elements of the alphabet of object-language of Σ+V.

A precise definition of the concept "*term* of Σ+V" is the following.

1) the above-mentioned *axiological variables* (see the definition of alphabet of Σ+V) are terms of Σ+V.

2) the above-mentioned *axiological constants* (see the definition of alphabet of Σ+V) are terms of Σ+V.

3) If Φ_k^n is an *n-placed axiological-value-functional symbol* (see the definition of alphabet of Σ+V), and $t_i, \ldots, t_n$ are *terms* of Σ-V, then $\Phi_k^n t_i, \ldots, t_n$ is a term of Σ+V. (It is worth noting here that signs $t_i, \ldots, t_n$ belong to the meta-language because they denote *any* terms of Σ+V. The analogous note is worth making with respect to the sign Φ_k^n belonging to the meta-language as well.)

4) If t_k is a term of Σ+V, then the expressions $\overleftarrow{t_k}$ and $\overrightarrow{t_k}$ are terms of Σ+V.

5) If t_k is a term of Σ+V, then the expression ®t_k is a term of Σ+V.

6) If t_k is a term of Σ+V, then the expression $/t_k/$ is a term of Σ+V.

7) If t_k is a term of Σ+V, then the expression $-/t_k/$ is a term of Σ+V.

8) An expression of the object-language of Σ+V is a term of Σ+V, then and only then, when it is so due to the above-formulated items 1)-7) of the given definition.

Thus, the *syntaxis* aspect of the abstract notion "*term* of Σ+V" is quite fixed. Now we are to move to constructing exact definition of the *syntaxis* aspect of the abstract notion "*formula* of Σ+V". To perform this move, let us accept the convention that in the given article, lowercase Greek letters α, β, and ω (belonging to meta-language) denote *any* formulae of Σ+V. Keeping this convention in mind, it is possible to give the following precise definition of the notion "formula of Σ+V".

1) All the propositional letters belong to the set of formulae of Σ+V.

2) When α and β are formulae of Σ+V, then all the expressions of the object-language of Σ+V, which (expressions) have forms $\neg\alpha$, $(\alpha \leftrightarrow \beta)$, $(\alpha \supset \beta)$, $(\alpha \vee \beta)$, $(\alpha \,\&\, \beta)$, belong to the set of formulae of Σ+V as well.

3) When t_i is a term of Σ+V, then $[t_i]$ is a formula of Σ+V.

4) When t_i and t_k are terms of Σ+V, then $(t_i =+= t_k)$ is a formula of Σ+V.

5) When t_i and t_k are terms of Σ+V, then $(/t_i/ = /t_k/)$ is a formula of Σ+V.

6) When t_i and t_k are terms of Σ+V, then $(/t_i/ = -/t_k/)$ is a formula of Σ+V.

7) When α is a formula of Σ+V, and the symbol Ψ (belonging to the meta-language) denotes any modality symbol from the set of {$\Box$, K, A, E, S, T, F, P, D, C, G, W, O, B, U, J, E}, then any expression of object-language of Σ having the form Ψα, is a formula of Σ+V also. It is worth noting here, that, strictly speaking, the expression Ψα (belonging to the meta-language) is not a formula of Σ+V, but a scheme of formulae of Σ+V.

8) Chains of symbols from the alphabet of object-language of Σ+V are formulae of Σ+V, if and only if it is so due to the items 1)-7) of the given definition.

In this part of the article which (part) is reduced intentionally to *syntaxis* of object-language of *multimodal* formal theory Σ+V, the set of modality symbols {$\Box$, K, E, A, S, T, F, P, D, C, G, W, O, B, U, J, E} is nothing but a set of very short *names*. The symbol $\Box$ is a name for the alethic modality "it is *necessary* that…". The symbols K, E, A, S, T, F, P, D, C, respectively, are names of/for the modal expressions "agent *Knows* that…", "agent *Empirically* (*a-posteriori*) *knows* that…", "agent *a-priori knows* that…", "under some concrete conditions in some definite time-and-space, an agent has a *Sensation*, *i.e. verification by feeling* (either immediately or by means of mediating tools), that…", "it is *True* that…", "agent has *Faith* that…(or agent believes that…)", "it is *Provable* in a consistent theory that…", "there is *an algorithm for Deciding* that…(hence, a machine could be constructed for such Deciding)", "it is *Consistent* that…".

The symbols G, W, O, B, U, J, E, respectively, are names of/for the modal expressions "it is *Good* (morally perfect) that…", "it is *Wicked* (morally bad, imperfect) that…", "it is *Obligatory* (mandatory, compulsory) that…", "it is *Beautiful* (aesthetically perfect) that…", "it is *Useful* (helpful, valuable, gainful, rewarding) that…", "it is a *Joy* (delight, happiness, pleasure) that…", "the state of affairs indicated and described by the dictum…, exists (or does exist)". In the present section of the article, pure syntaxis meanings of the modal symbols are defined quite precisely (although not manifestly but indirectly) by the below-given schemes of own (proper) axioms of multimodal formal philosophy (epistemology-and-axiology) system Σ+V which axioms are added to the ones of classical logic of propositions.

Thus, proper formal logic axioms and formal logic inference rules of Σ, Σ+C, and Σ+V are the ones of classical sentential logic calculus. Schemes of axioms and inference-rules of the classical propositional logic are applicable to all formulae of these three multimodal theories. Hence, the proper logic foundations of Σ, Σ+C, and Σ+V are identical but the mentioned logically formalized axiomatic systems based on these identical logic foundations are substantially different. It seems that, corresponding definitions of Σ, Σ+C, and Σ+V are identical, but strictly speaking, it only seems so. The formal theories Σ, Σ+C, and Σ+V have different alphabets of their object-languages, different sets of expressions, different sets of terms, different sets of formulae, different sets of definitions, different sets of axioms, and, finally, different sets of theorems.

In the given section of the article, exactly *syntax* meanings of all the modality symbols and of all the other special signs included into the alphabet of object language of Σ+V are defined precisely by the following list of schemes of proper philosophical (epistemological and axiological) axioms of Σ+V. (Certainly, such *axiomatic* definition of proper epistemology-and-axiology notions is *not manifest* one, but, nevertheless, it is *quite precise* one.) If α, β, ω are any formulae of Σ+V, then any such and only such expressions of the object language of Σ+V, which have the following forms, are *proper axioms* of Σ+V.

AX-1: $A\alpha \supset (\Box\beta \supset \beta)$.

AX-2: $A\alpha \supset (\Box(\omega \supset \beta) \supset (\Box\omega \supset \Box\beta))$.

AX-3: $A\alpha \leftrightarrow (K\alpha \,\&\, (\neg\Diamond\neg\alpha \,\&\, \neg\Diamond S\alpha \,\&\, \Box(\beta \leftrightarrow \Omega\beta)))$.

AX-4: $E\alpha \leftrightarrow \left(K\alpha \,\&\, \left(\Diamond\neg\alpha \vee \Diamond S\alpha \vee \neg\Box(\beta \leftrightarrow \Omega\beta)\right)\right)$.

AX-5: $\Omega\alpha \supset \Diamond\alpha$. (It is a nontrivial *multimodal generalization* of "Kant principle" combining the alethic and the deontic modalities: $O\alpha \supset \Diamond\alpha$.)

AX-6: $(\Box\beta \,\&\, \Box\Omega\beta) \supset \beta$. (It is a nontrivial *multimodal generalization* of the famous formula ($\Box\beta \supset \beta$) underivable in Σ+V. About the underivability of ($\Box\beta \supset \beta$), see paper [55].)

AX-7: $(t_i =\!+\!= t_k) \leftrightarrow \left(G[t_i] \leftrightarrow G[t_k]\right)$.

AX-8: $(t_i =\!+\!= g) \supset \Box G[t_i]$.

AX-9: $(t_i =\!+\!= b) \supset \Box W[t_i]$.

AX-10: $(G\alpha \supset \neg W\alpha)$. This axiom scheme is justified in A.A. Ivin's book [56].

AX-11: $(W\alpha \supset \neg G\alpha)$. This axiom scheme is justified in [56] as well.

AX-12: $A\alpha \supset \left(\left(\Phi\copyright xy =\!+\!= \circledR\Phi\copyright xy\right) \leftrightarrow \left(\left[\overleftarrow{\Phi\copyright xy}\right] \leftrightarrow \left[\overrightarrow{\Phi\copyright yx}\right]\right)\right)$.

AX-13: $A\alpha \supset \left(\left(\Phi\copyright xy =\!+\!= \circledR\Phi\copyright xy\right) \leftrightarrow \left(/\overleftarrow{\Phi\copyright xy}/ = -/\overrightarrow{\Phi\copyright yx}/\right)\right)$.

Definition DF-1 of the operation ® called "contraposition": 1) (($\circledR\overleftarrow{t_i} =\!+\!= \overrightarrow{\circledR t_i}$) & ($\circledR\overrightarrow{t_i} =\!+\!= \overleftarrow{\circledR t_i}$)) & ($\circledR\circledR t_i =\!+\!= t_i$), where t_i is a term of Σ+V; 2) $\circledR\Phi^2xy =\!+\!= \Phi^2\circledR y\circledR x$, where the symbol Φ^2xy stands for any binary operation; 3) $\circledR\Phi^1x =\!+\!= \copyright\Phi^1x$; (d) $\copyright\copyright\Phi^1x =\!+\!= \Phi^1x$, where Φ^1x stands for any unary operation.

Definition DF-2: if ω is a formula of Σ+V, then $\Diamond\omega$ is a *name* of/for $\neg\Box\neg\omega$.

In AX-3, AX-4, AX-5, and AX-6, the sign Ω (belonging to the meta-language) denotes only a "*perfection* modality". However, not all the modalities mentioned in this paper are called "*perfection* ones". For example, W and ◊ stand for modalities which are not perfections. In the present article, the set Δ of signs denoting *perfection*-modalities is {K, D, F, C, P, E, J, T, B, G, U, O, □}. Evidently, Δ is only a subset of the set of symbols denoting modalities mentioned in this paper.

In AX-12 and AX-13, the sign Φ (belonging to the meta-language) denotes a *binary* operation of the two-valued algebraic system of formal axiology, and the sign © (also belonging to the meta-language) denotes any such *one-placed* function, a value of which is the *inversion* of value of its variable.

Evidently, the precise *syntactic* definitions given above are *meaningless* from the *semantic* viewpoint. This is not an accidental result of negligence but such a consciously accepted scientific abstraction which is perfectly rational under some definite condition. Nontheless, for making the present paper perfectly meaningful one, now, it is upportune immediately to move to *semantics* of the artificial language of Σ+V.

2.3. An Unknown Semantics of/for the above-Defined Syntaxis of Object-Language of Formal Axiomatic Theory "Sigma+V"

Above in section 2.1 of this article, the definition of Σ+V is performed in the purely syntactic manner as the submitted formulation of Σ+V is ideliberately deprived of its concrete contents due to the relevant scientific abstraction. Hence, the multimodal axiomatic theory Σ+V is defined above and discussed hitherto as namely formal one. Below in this section of the article, I move from the pure syntaxis aspect of artificial language of Σ+V for defining its semantics.

The artificial language of Σ+V includes the well-known proper logic symbols and technical signs of classical mathematical logic of propositions. There is no need to provide here the habitual definitions of semantic meanings of the well-known propositional logic symbols as their habitual semantic meanings are already defined exactlly in relevant handbooks on mathematical logic. In particular, semantic meanings of the *sentential variables* (represented in Σ+V by the lowercase Latin letters "q", "p", "d", and also by the same letters having lower number indexes) are well-defined in available handbooks on classical sentential logic as well. However, it is necessary to provide definitions of semantic meanings of the unhabitual symbols (sometimes even odd compound ones) belonging to the artificial object-language of Σ+V.

Definition of semantic meanings is definition of an *interpretation-function*. For defining the interpretation-function it is necessary to define 1) a set called "domain (or realm) of interpretation" (let the letter Θ be used for denoting the interpretation domain) and 2) an evaluation-maker called "evaluator" $\mathcal{E}$. By definition, the set Θ (which is necessary for any standard interpretation of Σ+V), is such a set, every element of which possesses: 1) one and only one *axiological value* from the set {good, bad}; 2) one and only one *ontological value* from the set {exists, not-exists}.

The *axiological variables* (z, x, y, z_i, x_k, y_m) take their values from the set Θ.

The *axiological constants* "b" and "g" mean the *values* "bad" and "good", respectively.

Certainly, any concrete evaluation necessarily implies existence of a concrete evaluator (interpreter). Making an evaluation of an element from the interpretation-domain Θ by quite a definite (fixed) evaluator $\mathcal{E}$ is attaching an *axiological value* (good or bad) to the element. The valuator $\mathcal{E}$ is either any individual or any collective—it does not matter. Obviously, changing $\mathcal{E}$ may result in changing some valuations (relative ones), nevertheless, no change of valuator can change the set of laws of the algebraic system of formal axiology as these laws are not relative but absolute evaluations. By definition, the laws of two-valued algebra of formal axiology are such and only such *constant evaluation-functions* which possess the value g (good) under any possible combination of axiological values of their axiological variables. Certainly, $\mathcal{E}$ is a variable. It takes its values from the set of various evaluators. However, if an interpretation of Σ+V is well-defined, then the value of the variable $\mathcal{E}$ is well-defined also. Changing the value of $\mathcal{E}$ is changing the interpretation.

In the present article, "e" and "n" stand for "…exists" and "…not-exists", respectively. The signs "e" and "n" are named "*ontological constants*". By definition, in a standard interpretation of Σ+V, one and only one element of the set {{g, e}, {g, n}, {b, e}, {b, n}} corresponds to every element of Θ. The signs "e" and "n" belong to the meta-language. By definition of the alphabet of object-language of Σ+V, "e" and "n" do not belong to the object-language. Nevertheless, "e" and "n" are *indirectly* represented at the level of object-language of Σ+V by means of *square-bracketing*: "t_i exists" is represented by [t_i]; "t_i does not exist" is represented by ¬ [t_i]. This means that square-bracketing is a significant part of exact defining formal-axiological-and-ontological semantics of Σ+V.

N-placed terms of Σ+V are interpreted as such *n*-placed valuation-functions which are defined on the set Θ (the domain of interpretation). The notion "one-placed evaluation-function" is exemplified by the above-located **Table 1** and **Table 2**. The notion "*two-placed evaluation-function*" is instantiated by the above-located **Table 3**.

To reduce possibilities of misunderstanding, it is worth emphasizing here that in standard interpretations of Σ+V, the symbols M^1x, N_1^1x I_3^2xy, C^2xy, K^2xy, R^2xy denote not predicates but *evaluation-functions*. When a standard interpretation of Σ+V is fixed, then the expressions of object-language of Σ+V, which have forms (t_i =+= g), (t_i =+= b), (t_i =+= t_k), represent *predicates* in Σ+V.

According to the definition of semantics of the formal theory Σ+V, in a standard interpretation of this formal theory, a formula possessing the form (t_i =+= t_k) represents an either true or false proposition having the form "t_i is *formally-axiologically equivalent* to t_k" which proposition is true, if and only if the terms t_i and t_k possess identical axiological values (either "good" or "bad") under any combination of axiological values of the variables.

According to the definition of semantics of the formal theory Σ+V, in a fixed standard interpretation of this formal theory, a formula possessing the form (t_i =+= b) represents a proposition (either true or false one) having the form "t_i is a *formal-axiological contradiction*" which proposition is true, if and only if (in the fixed interpretation) the term t_i possess the value "bad" under any combination of axiological values of the variables.

According to the definition of semantics of the formal theory Σ+V, in a fixed standard interpretation of this formal theory, a formula possessing the form (t_i =+= g) represents a proposition (either true or false one) having the form "t_i is a *formal-axiological law*" (or "t_i is *absolutely good*") which proposition is true if and only if (in the fixed interpretation) the term t_i has the *value* "good" under any combination of the values of variables.

According to the definition of semantics of Σ+V, when t_i is a term in Σ+V, then, in a fixed interpretation of Σ+V, such a formula of Σ+V, which has the form [t_i], represents a *proposition* possessing the form "t_i exists" which proposition is *either true or false* one (in the fixed interpretation). By the definition, a formula having the form [t_i] is true in a fixed interpretation, if and only if t_i has the *ontological value* "e (exists)" in that fixed interpretation. By the definition, a formula having the form [t_i] is false in a fixed interpretation (of Σ+V), when and only whwn t_i has the ontological value "n (not-exists)" in that fixed interpretation.

Existence of expressions possessing the forms $\left[\overleftarrow{t_k}\right]$, $\left[\overrightarrow{t_k}\right]$, $/t_k/$, $-/t_k/$, $/\overleftarrow{t_k}/$, $/\overrightarrow{t_k}/$, $-/\overrightarrow{t_k}/$, $(/t_i/ = /t_k/)$, $(/\overleftarrow{t_i}/ = /\overrightarrow{t_k}/)$, $(/\overleftarrow{t_i}/ = -/\overrightarrow{t_k}/)$, in the object-language of Σ+V, makes up a very important qualitative difference between Σ and Σ+V. In relation to the expressions possessing such forms, herein, it is necessary to say that, in Σ+V, by definition of its object-language semantics, "/…/" means a *quantity magnitude* of "…", and "-/…/" denotes a *negative quantity magnitude* of "…".

Also, herein, it is necessary to say that, by definition of semantics of object-language of Σ+V, in a fixed standard interpretation of Σ+V, the symbol "=" denotes the *identity of quantity magnitudes*, consequently, $(/t_i/ = /t_k/)$ denotes *identity of quantity-magnitudes* of t_i and t_k. Hence, if and only if a concrete standard interpretation is well-fixed, then (in this well-defined interpretation) the formula $(/\overleftarrow{t_i}/ = -/\overrightarrow{t_k}/)$ represents a predicate (having a sufficiently definite truth-value in this interpretation). Herein, I presume that the syntactic and semantic meanings of the signs from the alphabet of formal arithmetic (for instance, the symbol "=") belonging also to the alphabet of object-language of Σ+V, are already well-defined in arithmetic [57]. Hence, it is not necessary to repete defining them manifestly in the given paper. In this relation, it is quite sufficient to make the reference to the well-written handbook "Introduction to Mathematical Logic" by E. Mendelson [57]. Therefore, the proper arithmetic axioms precisely definining the meanings of the above-exploited proper arithmetic signs ("=" and "-") were not included manifestly into the above-given definition of Σ+V, but it is presumed that these arithmetic axioms also belong to Σ+V.

The principal qualitative difference between Σ and Σ+V is created by the *vector* symbols "→" and "←" belonging to the alphabet of object-language of Σ+V and making the theory Σ+V significantly richer. The theories Σ and Σ+C deal with only *scalar* evaluation-functions. In contrast to/with them, being more general and fundamental, Σ+V operates with both the *scalar* and the *vector* functions. This is so because, owing to the definition of semantics of Σ+V, generally speaking, the evaluation-functions may be either *vectored* or *not-vectored* (*scalar*) ones. The above-given **Table 1**, **Table 2**, and **Table 3** exemplify *scalar* evaluation-functions. Due to exactly definining the semantics of object-language of Σ+V, the sign $\overleftarrow{t_i}$ denotes a *vectored* evaluation-function, which is a "synthesis" (combination or conjunction) of the scalar evaluation-function (t_i) and the proper vector (own inner direction) of t_i (indicated by "←"). Certainly, sometimes, either an evaluation-function does not possess a proper vector at all, or the inner (own) vector of evaluating is not important and, hence, may be ignored ("annihilated" by means of a quite acceptable abstraction). However, sometimes, a proper vector is significant and, consequently, ignoring it is a grave blunder (accepting the abstraction is not well-grounded). Exactly in this theoretically interesting and practically significant special case, using the rich language of Σ+V is quite relevant and meaningful. While such using the language of

Σ+V, the symbol "$\overleftarrow{t_i}$" denotes the proper vector (own inner direction) of t_i and the symbol "$\overrightarrow{t_i}$" denotes the *directly opposite* vector of t_i. Obviously, from the vewpoint of ordinary (statistically normal) human beings, the talk of vectored evaluation-functions is odd (unhabitual) one. Taking this talk seriously is a challenge to the dominating paradigm in the humanities to which formal philosophical ontology, axiology, logic, and philosophy of physics belong somehow (at least partly). Nevertheless, in contemporary theoretical physics (which could be evaluated as a necessarily mathematized part of the proper rational philosophy of nature), in principle, an abstract talk of vectored functions is not an unhabitual (odd) event. Modern physicists are used to systematical talking of vectors. Thus, in principle, statements of vectors in rational philosophy of nature can be meaninful and disputable. In the given article (namely, in its immediately following section), I apply the precedent made in mathematized theoretical physics to investigating the *substantially similar* (*analogous*) case of proper philosophical (formal-axiolofical) discourse of the classical mechanics. The results of application of the precedent are submitted below.

3. Results

The below-located finite sequence of formulae and schemes of formulae is an accomplishment of the principal goal of this article, namely, a *formal deductive inference* (within the axiomatic theory Σ+V) of such formulae which represent Galileo Galilei principle of relativity of motion, when a relevant physical interpretation of the formal theory Σ+V is given. The formal logical inference depends essentially of the three deliberately accepted nontrivial *assumptions* inserted manifestly into the below-located finite sequence. The first assumption is the one of *a-priori*-ness of knowledge. In Σ+V, this assumption is modeled by Aα. The second assumption used substantially in the below-located formal logical *inference* is a *formal-axiological analog* of Galileo Galilei principle of relativity of motion. This *formal-axiological analog* of the principle of motion relativity is modeled (within the formal theory Σ+V) by formula $R^2M^1xy =+= R^2M^1yx$. Another *formal-axiological* equation, namely, formula $R^2Q^1M^1xy =+= R^2Q^1M^1yx$ is also a *formal-axiological analog* of the principle by Galileo Galilei. The two *formal-axiological equations* (justified above within the algebraic system of formal axiology by computing relevant functions) have been already recognized as the *formal-axiological analogues* of Galileo Galilei principle of motion relativity, in [46]. But the below-located perfectly novel formal deductive derivation from the above-formulated nontrivial assumptions has not been invented, examined, and discussed hitherto.

1) $A\alpha \supset \left((\Phi©xy =+= ®\Phi©xy) \leftrightarrow \left(\left[\overleftarrow{\Phi©xy} \right] \leftrightarrow \left[\overrightarrow{\Phi©yx} \right] \right) \right)$: the axiom-scheme AX-12.

2) $A\alpha \supset \left((R^2M^1xy =+= ®R^2M^1xy) \leftrightarrow \left(\left[\overleftarrow{R^2M^1xy} \right] \leftrightarrow \left[\overrightarrow{R^2M^1yx} \right] \right) \right)$: from 1,

by substitution of R^2 for Φ, and substitution of M^1 for ©.

3) Aα: the *assumption* of a-priori-ness of knowledge.

4) $\left(\left(R^2M^1xy =+= ®R^2M^1xy\right) \leftrightarrow \left(\left[\overleftarrow{R^2M^1xy}\right] \leftrightarrow \left[\overrightarrow{R^2M^1yx}\right]\right)\right)$: from 2 and 3 by *modus ponens*.

5) $\left(\left(R^2M^1xy =+= ®R^2M^1xy\right) \supset \left(\left[\overleftarrow{R^2M^1xy}\right] \leftrightarrow \left[\overrightarrow{R^2M^1yx}\right]\right)\right)$: from 4 by the logic derivation rule called "elimination of $\leftrightarrow$".

6) $\left(R^2M^1xy =+= ®R^2M^1xy\right)$: the *assumption* justified in the algebraic system of formal axiology by computing evaluation-functions.

7) $\left(\left[\overleftarrow{R^2M^1xy}\right] \leftrightarrow \left[\overrightarrow{R^2M^1yx}\right]\right)$: from 5 and 6 by *modus ponens*.

8) $A\alpha, \left(R^2M^1xy =+= ®R^2M^1xy\right) \vdash \left(\left[\overleftarrow{R^2M^1xy}\right] \leftrightarrow \left[\overrightarrow{R^2M^1yx}\right]\right)$: by the succession 1—7. (Here, "$\ldots\vdash\ldots$" means "from…it is logically derivable in Σ+V, that…".)

9) $A\alpha \supset \left(\left(\Phi©xy =+= ®\Phi©xy\right) \leftrightarrow \left(/\overleftarrow{\Phi©xy}/ = - / \overrightarrow{\Phi©yx}/\right)\right)$: the axiom-scheme AX-13.

10)

$A\alpha \supset \left(\left(R^2Q^1M^1xy =+= ®R^2Q^1M^1xy\right) \leftrightarrow \left(/\overleftarrow{R^2Q^1M^1xy}/ = - / \overrightarrow{R^2Q^1M^1yx}/\right)\right)$: from 9, by substitution: of R^2 for Φ, and substitution of Q^1M^1 for ©.

11) $\left(\left(R^2Q^1M^1xy =+= ®R^2Q^1M^1xy\right) \leftrightarrow \left(/\overleftarrow{R^2Q^1M^1xy}/ = - / \overrightarrow{R^2Q^1M^1yx}/\right)\right)$: from 10 and 3 by *modus ponens*.

12) $\left(\left(R^2Q^1M^1xy =+= ®R^2Q^1M^1xy\right) \supset \left(/\overleftarrow{R^2Q^1M^1xy}/ = - / \overrightarrow{R^2Q^1M^1yx}/\right)\right)$: from 11 by the logic derivation rule called "elimination of $\leftrightarrow$".

13) $\left(R^2Q^1M^1xy =+= ®R^2Q^1M^1xy\right)$: the *assumption* justified in the algebraic system of formal axiology by computing evaluation-functions.

14) $\left(/\overleftarrow{R^2Q^1M^1xy}/ = - / \overrightarrow{R^2Q^1M^1yx}/\right)$: from 12 and 13 by *modus ponens*.

15) $A\alpha, \left(\left(R^2Q^1M^1xy =+= ®R^2Q^1M^1xy\right) \vdash \left(/\overleftarrow{R^2Q^1M^1xy}/ = - / \overrightarrow{R^2Q^1M^1yx}/\right)\right)$: by the succession 1-14.

Thus, the formal logical inference from the assumptions in Σ+V, is constructed. Now, the formal inference is to be interpreted. (Perhaps, here, it is worth recalling that, in the algebraic system of formal axiology, the symbol R^2xy stands for the noncommutative binary operation represented in natural human language by expressions "*x relative to y*", or "*x in relation to y*".) In accordance with the main target of the present article, R^2M^1xy is interpreted herein as the evaluation-function "*movement* of *x in relation to y*". R^2M^1yx is interpreted as the evaluation-function "*movement* of *y in relation to x*". $R^2Q^1M^1xy$—the evaluation-function "*quickness of movement* of *x in relation to y*". $R^2Q^1M^1yx$—"*quickness of movement* of *y in relation to x*". Evaluation-functional meanings of the signs M^1, Q^1, R^2 are precisely defined above (in 2.1) by **Table 1**, **Table 2**, and

Table 3, respectively. By means of the exact definitions given above in this paper, it is possible to justify quite convincingly (by attentive calculating compositions of relevant valuation-functions) that (R^2M^1xy =+= R^2M^1yx) and, consequently, according to the above-given definition of ®, it is true that (R^2M^1xy =+= ®R^2M^1xy). These two equations mean that (scalar aspect of) *movement of x in relation to y* is *formally-axiologically equivalent* to (scalar aspect of) movement of *y in relation to x*.

Concerning *quickness* (*speed*) of movement, it is relevant to affirm here that due to the above-given exact definitions, it is possible to justify quite convincingly (by attentive computing compositions of appropriate valuation-functions) that ($R^2Q^1M^1xy$ =+= $R^2Q^1M^1yx$) and, consequently, according to the above-given definition of ®, it is true that ($R^2Q^1M^1xy$ =+= ®$R^2Q^1M^1xy$). These two equations modeling the *speed* (*rapidity*) *aspect* of Galileo's principle of relativity of motion mean that *scalar aspect of velocity* (*i.e. quickness*, *speed*) of *movement of x in relation to y* is *formally-axiologically equivalent* to *scalar aspect of velocity* (quickness, rapidity) of movement of *y in relation to x*. Originally, the surprising *formal-axiological* identifications of the compositions of valuation-functions relevant to Galileo's relativity principle have been discovered (recognized) or invented (constructed on purpose) in the article [46], in which article either of the wonderful identifications has been named "a *formal-axiological law of contraposition* of R^2xy".

In the mentioned original paper [46], also for the first time, a *vector interpretation* of R^2xy has been suggested and investigated as well. This means that, in [46], for the first time, the *formal-axiological law of contraposition* of R^2xy has been interpreted as a namely *vectored* law of contraposition. Thus, for the first time, the *formal-axiological analogs* of Galileo's principle of relativity of *vectored* motion have been created (noticed or invented) and demonstrated (by accurate computing appropriate valuation-functions) in [46]. But the discovered (noticed) or created on purpose (in [46]) *formal-axiological analogs* are not affirmations of "what is". They are affirmations of "what is good". According to the principle ascribed to D. Hume by interpreters of his famous treatise [58] and also according to G.E. Moore's systematical anti-naturalism in ethics [59], a logically unbridgeable gap exists between "what is" and "what is good". Moreover, according to that intellectually respectable empiricist paradigm which still dominates in the humanities, formal logical bridging the gap between "is good" and "is" is absolutely impossible.

Thus, although, precise formulating and algebraic justifying *the formal-axiological analog* of Galileo Galilei principle of relativity of motion had been discovered (invented) and published initially in [46], the above-mentioned notorious formal-logic gap had not been bridged logically in [46], and the firm faith that the gap is logically unbridgeable had not been challenged in [46]. In contrast (and in substantial supplement) to [46], for the first time in relevant literature, the present article has submitted an option of logical bridging the allegedly unbridgeable formal-logic gap between the relativity principle by Galileo Galilei and its already discovered and published formal-axiological analog. The above-submitted logical bridging is the main nontrivial scientific innovation of the present paper. It is a significant challenge to the habitual paradigm limiting development of science.

The substantially novel nontrivial result submitted in the present article means that *under some quite definite extraordinary condition*, there is a possibility of formal logical bridging the gap between "is" and "is good". The belief that the gap is logically unbridgeable is adequate if and only if knowledge is *empirical*. Consequently, if and only if knowledge is *a priori*, the gap is logically bridgeable. According to the above-said, from conjunction of the assumption of *a-priori*-ness of knowledge, and the two formal-axiological equations grounded deductively in algebra of formal axiology, Galileo's principle of relativity of motion is formally logically derived in Σ+V.

The result of interpretation and translation of formula $\left(\left[\overleftarrow{\mathrm{R}^2\mathrm{M}^1\mathrm{xy}}\right] \leftrightarrow \left[\overrightarrow{\mathrm{R}^2\mathrm{M}^1\mathrm{yx}}\right]\right)$ from the artificial language into the natural one of human beings is the following: "A vectored movement of *x in relation to y* exists if and only if the oppositely vectored movement of *y in relation to x* exists". The result of interpretation and translation of formula $\left(/\overleftarrow{\mathrm{R}^2\mathrm{Q}^1\mathrm{M}^1\mathrm{xy}}/ = -/\overrightarrow{\mathrm{R}^2\mathrm{Q}^1\mathrm{M}^1\mathrm{yx}}/\right)$ from the artificial language into the natural one of human beings is the following: "quantity-magnitude of quickness of vectored movement of *x in relation to y* is exactly equal to the negative quantity-magnitude of quickness of vectored movement of *y in relation to x*".

4. Discussion

4.1. Representing Relativity in a Discrete Mathematical Model of Formal Philosophical Ontology in General and of Antiquity Physics Especially: Being and Nonbeing, Noncontradiction and Movement, Being and Consistency of the World, Material World and Universal Interconnection (as Evaluation-Functions in Algebra of Formal Axiology)

Above in this article, only such two *formal-axiological equations* 4) and 5) of two-valued algebraic system of formal axiology have been considered, which are necessary premises (manifestly accepted basic assumptions) of/for the main result of the paper, namely,—formal logical deductive inference of Galileo Galilei relativity principle in the formalized axiomatic epistemology-and-axiology theory Σ+V, under the condition of *a-priori*-ness of knowledge. Now let us continue generating and discussing *formal-axiological equations* of the two-valued algebraic system of formal axiology concerning 1) abstract philosophical theory of being in general and 2) the material world as subject-matter of proper physics especially. Certainly, some of the following *formal-axiological* equations making up the mathematical model are not directly connected with Galileo Galilei principle of relativity of motion but they are quite relevant to physics in general and to physics of Antiquity especially. (Herein, first of all, I imply the early Greek physicists of Ionia [60] [61], "Physics" by Aristotle [62], etc.) Nevertheless, along with construction and discussion of the discrete mathematical model of philosophical ontology (as formal axiology) in general, the formal-axiological meaning of the word "relativity" as an evaluation-function (x relative to y) in two-valued algebra of formal axiology is defined precisely and studied especially. Thus, being focused mainly on the concrete meaning of the word "relativity" used by Galileo Galilei, the given paper is targeted also at recognizing and defining precisely an abstract *universal* meaning of the word "*relativity* (of x to y)" which is *common for* meanings of "relativity" in morals [58] [63] [64] and in physics [13] [14] [15] [43] [44] [65] [66].

To continue generating and discussing the list of equations making up the discrete mathematical model under construction and investigation, by means of the following glossary for below-placed **Table 4**, I introduce a set of new valuation-functions.

Glossary for the below-located **Table 4**. The symbol $P_1{}^1x$ stands for the value-function "*possibility* of x". $I_3{}^1x$—"*impossibility* of x". $G_1{}^1x$—"*genesis* (*appearance*, *emergence*, *creation*) *of* x". $G_2{}^1x$—"*genesis* (*appearance*, *emergence*, *creation*) *from* x". $D_5{}^1x$—"disappearing (dissolving) in (what, whom) x". $D_6{}^1x$—"x's disappearance (dissipation)". $C_1{}^1x$—"contradictoriness (*inconsistency*) *of* x". $C_2{}^1x$—"*consistency* (non-contradictoriness) *of* x". $C_3{}^1x$—"*contradiction* (*what*) x". $D_6{}^1x$—"*division* (*split*) *of* x, or *dividedness of* x". $M_4{}^1x$—"*measuring* (what, whom) x", or "*measurement of* (what, whom) *x as an object*". $M_5{}^1x$—"*measurement by* (what, whom) x". These valuation-functions are defined precisely by the following **Table 4**.

The discrete mathematical model under construction and discussion in this part of the paper is made up by the following chain (queue) of formal-axiological equations. The immediately following succession of equations begins with number 5) as it is a continuation of the above-started list of formal-axiological equivalences. (The list starts above in the section 2.1 of this article.)

5) $B_1{}^1x$ =+= $B_1{}^1x$: "being is", or "what is, is" (Parmenides [61] [67]).

6) $N_1{}^1x$ =+= $N_1{}^1x$: "nonbeing is not", or "what is not, is not" (Parmenides [61] [67]).

3) $B_1{}^1N_1{}^1x$ =+= $N_1{}^1B_1{}^1x$: being of nonbeing is nonbeing of being.

4) $B_1{}^1x$ =+= $N_1{}^1N_1{}^1x$: being of x is nonbeing of nonbeing of x.

5) $N_1{}^1x$ =+= $B_1{}^1N_1{}^1x$: nonbeing of x is being of nonbeing of x.

6) $N_1{}^1x$ =+= $N_1{}^1B_1{}^1x$: nonbeing of x is nonbeing of being of x.

7) $B_1{}^1x$ =+= $B_2{}^1x$: being of x is equivalent to being of x by itself.

8) $B_2{}^1x$ =+= B^2xx: being of x by itself is equivalent to x's being by x.

9) B^2xx =+= $B_1{}^1x$: being of x by x is equivalent to x's being.

10) $B_1{}^1x$ =+= x: being of x is equivalent to x.

11) $B_4{}^1x$ =+= C^2xx: being of x in itself is x's being in x.

Table 4. Defining the value-functions determined by one variable.

x	P_1^1x	I_3^1x	G_1^1x	G_2^1x	D_5^1x	D_6^1x	C_1^1x	C_2^1x	C_3^1x	$\mathrm{D}_6^1\mathrm{x}$	M_4^1x	M_5^1x
g	g	b	g	b	g	b	b	g	b	b	b	g
b	b	g	b	g	b	g	g	b	b	g	g	b

12) C^2xx=+= g: being of x in x is a *formal-axiological law* of two-valued algebra of metaphysics. By the way, this *formal-axiological* equation is quite relevant to the notorious stormy and long (even still existing yet) discussion of I. Kant's "being of *thing in itself*" [34] [35].

13) B_3^1x =+= $B^2xE_2^1x$: being of x by its nature is x's being by nature (essence) of x.

14) B_1^1x =+= $N_1^1M^1x$: being of x is equivalent to nonbeing of movement (change) of x (Parmenides, Zeno, Melissus [61] [67]).

15) M^1x =+= N_1^1x: movement (change) of x is equivalent to nonbeing of x (Parmenides, Zeno, Melissus [61] [67]).

16) C_3^1x =+= O^2xx: contradiction x is equivalent to self-contradiction to (or by) x.

17) C_3^1x =+= b: contradiction x is equivalent to the *negative value constant b*.

18) C^2xb =+= N_1^1x: being of contradiction in x is equivalent to nonbeing of x (Parmenides, Zeno, Melissus [61] [67], Aristotle [62]).

19) C_1^1x =+= C^2xb: inconsistency (contradictory-ness) of x means being of contradiction (self-contradiction) in x.

20) C_1^1x =+= N_1^1x: inconsistency (contradictory-ness) of x is equivalent to nonbeing of x.

21) B_1^1x =+= $N_1^1C^2xb$: being of x is equivalent to nonbeing of (being of contradiction in x).

22) M^1x =+= C_1^1x: movement (change) of x is equivalent to inconsistency (contradictory-ness) of x (Parmenides, Zeno, Melissus [61] [67]).

23) B_1^1x =+= C_2^1x: being of x is equivalent to consistency (noncontradictory-ness) of x (Parmenides, Zeno, Melissus [67], Aristotle [62]).

24) M_3^1x =+= N_1^1x: matter, material, materialness of x is equivalent to nonbeing of x. (Plato [68], Aristotle [62], Plotinus [69], W.K.S. Guthrie [70] [71] [72]).

25) M^1x =+= M_3^1x: movement (change) of x is equivalent to matter, material, materialness of x.

26) M_3^1x =+= C_1^1x: matter, material, materialness of x is equivalent to inconsistency (contradictory-ness) of x.

27) $M_3^1W^1x$ =+= $C_1^1W^1x$: materialness of world of x is equivalent to inconsistency (contradictory-ness) of world of x.

28) B_1^1x =+= G_1^1x: being of x is equivalent to genesis of x. (Consequently, in relation to the algebraic system under discussion, the words "being" and "genesis" are synonyms. Hence, both translations "The Book of *Being*" and "The Book of *Genesis*" are quite acceptable in this relation.)

29) G_1^1x =+= $G^2N_1^1xx$: genesis of *x* is genesis of *x* from nonbeing of *x*.

30) B_1^1x =+= $G^2N_1^1xx$: being of *x* is genesis of *x* from nonbeing of *x*.

31) B_1^1x =+= $P_1^1G^2N_1^1xx$: being of *x* is equivalent to possibility of genesis of *x* from nonbeing of *x*.

32) $I_3^1G^2N_1^1xx$ =+= N_1^1x: impossibility of genesis of *x* from nonbeing of *x* is equivalent to nonbeing of *x*.

33) B_1^1x =+= $P_1^1G^2N_1^1M_3^1W^1xM_3^1W^1x$: being of *x* is equivalent to possibility of genesis of material world of *x* from nonbeing of material world of *x*.

34) $I_3^1G^2N_1^1M_3^1xM_3^1x$ =+= $M_3^1W^1x$: impossibility of genesis of matter of *x* from nonbeing of matter of *x* is equivalent to materialness of world of *x*.

35) $M_3^1W^1x$ =+= $I_3^1G^2N_1^1M_3^1W^1xM_3^1W^1x$: materialness of world of *x* is equivalent to impossibility of creation (genesis) of material world of *x* from nonbeing of material world of *x*.

36) $M_3^1W^1x$ =+= $I_3^1G^2N_1^1xx$: materialness of world of *x* is equivalent to impossibility of genesis of *x* from nonbeing of *x*. This equation models the materialistic views of the well-known early Greek physicists of Ionia [60] [61]. In contrast (it seems that even in logical opposition) to the equations 34)-36) modeling the materialistic world-views by Ionia physicists, the equations 28)-33) model the religious ontology of creation (genesis) represented in Biblia. Herein, I imply "The Book of Being (Genesis)" especially. However, the impression of logical opposition is a logic-linguistic illusion as the alleged opposites belong to different worlds, namely, to the one of *being material* and to the one of *being proper* (in Parmenides' meaning of the word "being").

37) B_1^1x =+= $C^2M_3^1W^1xI_1^2zy$: being of *x* is equivalent to being of *universal interconnection* (of every *z* with every *y*) in the material world of *x* [51]. At the level of too vague and ambiguous natural language of humans, this profound ontological idea had been expressed somehow since ancient times to our days by many great philosophers, for example, by G.W. Leibniz ([73] pp. 116, 424).

38) B_1^1x =+= $C^2M_3^1W^1xI_2^2zy$: being of *x* is equivalent to being of *universal interrelation* (of any *z* with any *y*) in the material world of *x* [51].

39) B_1^1x =+= $C^2M_3^1W^1xI_6^2zy$: being of *x* is equivalent to being of *universal interdependence* (of any *z* and any *y*) in the material world of *x* [51].

40) B_1^1x =+= $C^2M_3^1W^1xI_4^2zy$: being of *x* is equivalent to being of *universal interaction* (between every z and every y) in the material world of *x* [51].

41) B_1^1x =+= $C^2M_3^1W^1xI_3^2zy$: being of *x* is equivalent to being of *universal inter-determination* in the material world of *x* [51].

42) M_3^1x =+= R_1^1x: materialness of *x* is equivalent to relativity of *x*.

43) R_1^1x =+= $O_1^1R_2^1x$: relativity *of x* is opposite to relativity *to x*.

44) R_1^1x =+= $O_1^1B_2^1x$: relativity of *x* is opposite to being of *x* by itself.

45) R_2^1x =+= B_2^1x: relativity *to x* is equivalent to being of *x* by itself.

46) R_2^1x =+= x: relativity *to x* is equivalent to *x*.

47) B_1^1x =+= R_2^1x: being of *x* is equivalent to relativity *to x*.

48) B_1^1x =+= $N_1^1R_1^1x$: being of *x* is equivalent to nonbeing of relativity of *x*.

49) $B_1{}^1x$ =+= $R_1{}^1R_1{}^1x$: being of x is equivalent to relativity of relativity of x.

50) $B_1{}^1x$ =+= $R_1{}^1N_1{}^1x$: being of x is equivalent to relativity of nonbeing of x.

51) $N_1{}^1R_1{}^1N_1{}^1x$ =+= $N_1{}^1x$: nonbeing of relativity of nonbeing of x is equivalent to nonbeing of x.

52) $B_1{}^1x$ =+= $R_1{}^1I_3{}^1x$: being of x is equivalent to relativity of impossibility of x.

53) $R^2I_3{}^1xy$ =+= $R^2I_3{}^1yx$: (impossibility of x) relative to y is *formally-axiologically equivalent* to (impossibility of y) relative to x.

54) $B_1{}^1x$ =+= $R_1{}^1C_1{}^1x$: being of x is equivalent to relativity of contradictoriness of x.

55) $B_1{}^1x$ =+= $R_1{}^1M_3{}^1x$: being of x is equivalent to relativity of matter (materialness) of x.

56) $R^2M_3{}^1xy$ =+= $R^2M_3{}^1yx$: (x's being material) relative to y is *formally-axiologically equivalent* to (y's being material) relative to x.

57) $N_1{}^1R_1{}^1M_3{}^1x$ =+= $N_1{}^1x$: nonbeing of relativity of matter (materialness) of x is equivalent to nonbeing of x.

58) $R^2M_4{}^1xy$ =+= $R^2M_4{}^1yx$: ((measurement of x) relative to y) is *formally-axiologically equivalent* to ((measurement of y) relative to x).

59) $R^2D_6{}^1xy$ =+= $R^2D_6{}^1yx$: (x's being divided) relative to y is *formally-axiologically equivalent* to (y's being divided) relative to x.

60) $B_1{}^1x$ =+= $R_1{}^1M^1x$: being of x is equivalent to relativity of movement (change) of x.

61) $N_1{}^1R_1{}^1M^1x$ =+= $N_1{}^1x$: nonbeing of relativity of movement of x is equivalent to nonbeing of x.

A remarkable subset of the set of above-listed affirmations can be modeled concisely (given in a short economical form) by a significantly *more general* statement represented by the following theorem-scheme which condense the knowledge.

62) $R^2©xy$ =+= $R^2©yx$: ((©x) relative to y) is *formally-axiologically equivalent* to ((©y) relative to x). Herein, it is worth recalling that, in the given paper, the symbol © (belonging to the meta-language) stands for *any* one-placed function, values of which are opposites (inversions) of values of its argument. In some relation, theorem-scheme 62) is a substantial *generalization* of Galileo's principle of relativity of locomotion. The relativity of locomotion in mechanics is a well-known *particular* case of the hitherto unknown *more universal relativity principle* represented by 62) which may be called a "general law of *contraposition of relativity*". The above-formulated hitherto unknown "principle of *relativity of nonexistence*" (see equation 3 in section 2.1 of this article) is another noteworthy particular case of the law of *contraposition of relativity*.

Thus, the two-placed evaluation-function "*relativity* of x to y" and its particular (degenerate) cases, namely, "*relativity* of x" and "*relativity* to y" are indispensable for making such an adequate discrete mathematical model of philosophy of nature which (model) combines both the proper *axiological* and the proper *ontological* aspects of heavily mathematized system of strictly universal *pure a* *priori* principles of theoretical physics. By the way, the remarkable intellectual tendency to combine essentially the proper axiological and the proper ontological aspects has been developing since ancient to modern times, for example, since Anaximander of Miletus [60] ([61] pp. 89-92) and Plotinus [69] to G.W. Leibniz [73] and A.A. Lovelace [1], in spite of the positivists.

4.2. Recognizing and Exploiting a "Mole Hole" for Formal Logical Inferring "Is" from "Is-Good", and for Reverse Formal Logical Deriving "Is-Good" from "Is", within the Axiomatic Epistemology-and-Axiology Theory Σ+V for the Sake of Making Nontrivial Discoveries in Physics

Originally, the formal logical "mole hole" deductively bridging (under some *extraordinary* epistemic condition) the allegedly unbridgeable gap between "is" and "is-good" has been discovered (accidentally noticed or intentionally created—it does not matter) in [38]. Then, being quite recognized, the discovery (or invention) of formal-logical (deductive) "mole hole" has been systematically exploited on purpose in [40] [41] for philosophical grounding *pure a priori* knowledge of *strictly universal* principles of proper theoretical physics. Also in the present article, the "mole hole" has been used on purpose for axiomatic grounding the motion relativity principle by Galileo Galilei. According to the above-submitted investigation results, Galileo's principle of relativity of motion is strictly-logically (deductively) grounded in the formal axiomatic epistemology-and-axiology theory Σ+V, by means of the above-constructed formal deductive derivation (from the triple of manifestly indicated and well-defined nontrivial assumptions).

Along with the above-considered "mole hole" in Σ+V for formal-logical bridging such statements of "what is" and "what is good", which are affirmations of vectors, there is also a "mole hole" in Σ+V for formal-logical bridging such judgements of "what is" and "what is good", which have nothing to do with vectors. Firstly, I mean the psychologically surprising theorem-schemes $(A\alpha \supset (\alpha \leftrightarrow G\alpha))$ and $(A\alpha \supset (\Box\alpha \leftrightarrow G\alpha))$, which are formally-logically provable in the formal axiomatic theory Σ+V, along with the psychologically unexpected theorem-scheme $(A\alpha \supset (\alpha \leftrightarrow \Box\alpha))$. Formal proofs of these psychologically odd theorem-schemes are already published in [40] [41] [55] [74]. In the indicated triple of theorem-schemes (which are the implications), *generally* speaking, the consequents are false, but in that very rare (extraordinary) *particular* case, when it is true that Aα, the implications are true and formally provable in Σ+V. Secondly, I mean the wonderful theorem-scheme $(A\alpha \supset ((t_i =+= t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k])))$ a formal-logical proof of which in the formal theory Σ (having nothing to do with vectors) has been published originally in [38] [40] [41]. In the above-defined formal axiomatic theory Σ+V, the wonderful theorem-scheme $(A\alpha \supset ((t_i=+=t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k])))$ is also formally provable as all the axiom-schemes and logic-inference-rules, which are necessary and sufficient for formally proving it, belong to Σ+V as well.

Consequently, the following formal logical inference may be constructed in Σ+V.

1) $A\alpha \supset ((t_i =+= t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k]))$: the theorem-scheme.

2) $A\alpha$: assumption.

3) $(t_i =+= t_k) \leftrightarrow ([t_i] \leftrightarrow [t_k])$: from 1 and 2 by *modus ponens*.

4) $(t_i =+= t_k) \supset ([t_i] \leftrightarrow [t_k])$: from 3 by the rule of elimination of $\leftrightarrow$.

5) $(R^2M_4^1xy =+= R^2M_4^1yx) \supset ([R^2M_4^1xy] \leftrightarrow [R^2M_4^1yx])$: from 4, by substitution of $R^2M_4^1xy$ for t_i, and $R^2M_4^1yx$ for t_k.

6) $(R^2M_4^1xy =+= R^2M_4^1yx)$: premise (see equation # 58 in the above-generated list).

7) $([R^2M_4^1xy] \leftrightarrow [R^2M_4^1yx])$: from 5 and 6 by *modus ponens*.

8) $A\alpha, (R^2M_4^1xy =+= R^2M_4^1yx) \vdash ([R^2M_4^1xy] \leftrightarrow [R^2M_4^1yx])$: by 1-7.

A translation of $([R^2M_4^1xy] \leftrightarrow [R^2M_4^1yx])$ into the natural language of humans is the following: measuring x in relation to y is measuring y in relation to x. Another translation: measurement of x relative to y takes place, if and only if measurement of y relative to x takes place. This means that *measurement is relative*. And this is a statement of what is. The statement may be called "the *principle of relativity of measurement*". There is a very important fundamental analogy between this principle and Galileo's principle of relativity of movement, as both evaluation-functions "measurement of x" and "movement of x" are *inversions* of x's value. Moreover, the above-submitted succession 1-8 can be continued as follows.

9) $(M_4^1x =+= M^1x) \supset ([M_4^1x] \leftrightarrow [M^1x])$: from 4, by substitution of M_4^1x for t_i, and of M^1x for t_k.

10) $(M_4^1x =+= M^1x)$: such a *premise* which can be justified by comparing the tabular definitions of M_4^1x and M^1x (see **Table 1** and **Table 4**, respectively).

11) $([M_4^1x] \leftrightarrow [M^1x])$: from 9 and 10 by *modus ponens*.

12) $A\alpha, (M_4^1x =+= M^1x) \mid\!\!— ([M_4^1x] \leftrightarrow [M^1x])$: by 1-11.

A translation of $([M_4^1x] \leftrightarrow [M^1x])$ into the natural language of humans is the following: *a measurement of x* takes place, if and only if a *change* of *x* takes place. Another translation: for any *x*, *measuring x* is *changing x*. In other words: *measuring is changing the object of measuring*. *This is a statement of what is*. The statement is formally-logically derived in Σ+V from the *formal-axiological* equation, *i.e*. from the *statement of values*, under the extraordinary epistemic condition that Aα.

Thus, in some *extraordinary* sense (taking into an account not only empirical but also *a priori* knowledge), Ada Lovelace was quite right when she wrote in her letter to Andrew Crosse: "There is too much tendency to making *separate* and *independent bundles* of both the physical and the moral facts of the universe. Whereas, all and everything is naturally related and interconnected" [1]. Certainly, in their *ordinary concrete* meanings, "*moral* relations and values" imply "*human* ones" which (according to the dominating worldviews of humans) do not exist among stones, planets, stars, and galaxies. Therefore, herein, we are to elevate (generalize) significantly the *ordinary* concrete meaning of "moral" (in natural human languages) to *extraordinary abstract* one of *artificial language of universal formal axiology* operating with *abstract values* having moral values of humans as modest particular cases of the necessarily existing *formal-axiological side* (aspect) of universe. Thus, the "mole hole" naturally connects the *axiological* side (aspect) of universe with its *ontological* one, under the precisely defined extraordinary condition.

4.3. Clarifying the Improvement of the Proposed Method and Explaining Why the Expectations Are Fulfilled

Herein, to clarify the improvement of the proposed method and to explain why the expectations are fulfilled while other methods cannot, I am to highlight the following items exhibited as bullet points.

1) The proper *empirical* methods (of observations and experiments) cannot justify perfectly (quite sufficiently) such *strictly universal* or necessarily universal (or necessarily necessary) statements of proper theoretical physics, which are statements of *pure a priori* knowledge of nature, *i.e.* statements of such an extraordinary knowledge of it which is *independent of any physical experience and exists before it*. That is why a discussion of physical *experience* proper (in particular, of *physical experiments*) is not included into this paper. This is so because a discussion of *experiments* is not relevant to the theme and to the main goal of the article intentionally reduced completely to improving *formal logical structure of the pure theory* of nature.

2) This explains why exactly *axiomatic* method has been systematically exploited in the present article, and the exploited logic is not inductive but *deductive* one. (Certainly, it is presumed herein that the traditional (not-mathematized) formal logic cannot be an effective method of/for realizing the goal of this paper, only the modern mathematical logic using artificial languages is an appropriate method for doing this).

3) I have improved the method of constructing and investigating logically formalized axiomatic theories by manifest including *formal-axiological* aspect to it. The result of improvement is synthetic *multimodal* one. The mathematical logic systems having nothing to do with modalities are not suitable for the synthetic goal. Even the modal logic systems dealing with *only one kind* of modalities are not appropriate (too primitive) for it. The sufficiently improved method for realizing the synthesis goal implies logical uniting epistemic, axiological, and some other kinds of modalities in one logically formalized axiomatic theory.

4) The proposed substantial improvement of the method of constructing and investigating logically formalized axiomatic theories has resulted in creating a possibility of invention (construction) of such multimodal axiomatic *epistemol-*

ogy-and-axiology system Σ+V, in which there is a possibility of invention (construction) of a *perfectly formal deductive inference* of Galileo's relativity principle from some nontrivial assumptions precisely defined in that multimodal axiomatic system.

5) For the first time in the world professional literature, the possibility of constructing the *formal logical inference* has been realized above in the given paper, namely, in section 3. Results. Consequently, the main expectations and the main results coincide. Thus, the above-formulated principal goal of this article is fulfilled.

5. Conclusions

Generally speaking, from the present article as a whole, it follows logically that under the *extraordinary* epistemic condition of *a-priori*-ness of knowledge, in the axiomatic *epistemology-and-axiology* system Σ+V, there is a *rare* possibility of exactly formal-logical deriving "what is" from "what is-good" (and conversely), *i.e*. the possibility of fundamental connecting ontology and axiology, respectively. It has been recognized and justified in the given article, that the impossibility of formal-logical bridging the gap between "is" and "is-good" is *not absolute but relative*. Certainly, the impossibility of logical (deductive) bridging the gap remains quite a universal principle for (and *only for*) the habitual (ordinary) domain of *empirical* knowledge, but the discovered "mole-hole" (significant *exclusion* from the allegedly universal principle) is located within the unhabitual (extraordinary) realm of (and *only of*) *pure a-priori* knowledge, *i.e*. beyond the domain of *empirical* knowledge. However, this abstract formulation of the qualitatively new attitude to the universal concept of relativity is very wide (extremely general), consequently, by means of this abstract conclusion formulation it is not easy to maintain the key findings of the given article. Therefore, in order to make the conclusion section more concrete and quite clear, below I include the point-by-point findings of this article.

1) A logically formalized multimodal axiomatic epistemology-and-axiology theory Σ+V has been exactly formulated (in artificial language) and precisely defined for the first time.

2) An original attempt of applying the formal theory Σ+V to the system of classical mechanics with a view of formalizing it logically has been undertaken.

3) In the logically formalized axiomatic theory Σ+V, a hitherto unknown formal logical derivation (inference) of the well-known Galileo principle of relativity of locomotion has been constructed for the first time.

4) The nontrivial assumptions, from which Galileo's relativity principle is formally logically derivable in Σ+V, are exactly formulated (in the artificial language of Σ+V) and precisely defined for the first time.

5) One of the three nontrivial assumptions, namely, the assumption Aα (of *a-priori-ness* of knowledge) is defined precisely (although indirectly) in Σ+V (by the system of its axioms and logic derivation rules) for the first time.

6) Also, for the first time other two of the three nontrivial assumptions, namely, the couple of *formal-axiological equations* of two-valued algebra of *formal axiology*, which couple makes up a *formal-axiological analog* (*model*) of Galileo's principle of relativity of motion, is represented in Σ+V. In the two-valued algebraic system of formal axiology, this couple of *formal-axiological equations* is exactly formulated and justified by accurate computation of compositions of evaluation-functions relevant to physics. Combining this couple of *formal-axiological* equations with the epistemic assumption of *a-priori*-ness of knowledge makes up such a triple of premises from which (triple) Galileo's relativity principle is formally derived in Σ+V (given the appropriate physical interpretation of the formal theory).

In general, the surprising formal logical derivation of Galileo's relativity principle (in the logically formalized multimodal axiomatic epistemology-and-axiology system Σ+V) means that proper philosophical foundations of proper theoretical physics contain not only ontological, epistemological, and formal-logical aspects, but also *formal-axiological* one, and the four aspects are necessarily connected somehow. Hence, inventing and investigating mathematical models of their essential connections are heuristically significant for proper theoretical physics and, therefore, worth undertaking. Who knows, probably, deductive logic justifications of some other strictly universal "mathematical principles of natural philosophy" also could be formally represented (modeled) in Σ+V (or successfully grasped as a result of its mutation). Let us wait and see.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

References

[1] Lovelace, A.A. (1844) A Letter to Andrew Crosse. https://mathshistory.st-andrews.ac.uk/Extras/Lovelace_letter

[2] Hilbert, D. (1990) Foundations of Geometry [Grundlagen der Geometrie]. Open Court Publishing Co., La Salle.

[3] Hilbert, D. (1996) Axiomatic Thought. In: Ewald, W.B., Ed., *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Vol. 2, Oxford University Press, Oxford, 1105-1115.

[4] Hilbert, D. (1996) The New Grounding of Mathematics: First Report. In: Ewald, W.B., Ed., *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Vol. 2, Oxford University Press, Oxford, 1115-1134.

[5] Hilbert, D. (1996) The Logical Foundations of Mathematics. In: Ewald, W.B., Ed., *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Oxford University Press, Oxford, 1134-1147.

[6] Hilbert, D. (1996) Logic and the Knowledge of Nature. In: Ewald, W.B., Ed., *From Kant to Hilbert: A Source Book in the Foundations of Mathematics*, Oxford University Press, Oxford, 1157-1165.

[7] Hilbert, D. (1996) On the Infinite. In: Ewald, W.B., Ed., *From Kant to Hilbert*: *A Source Book in the Foundations of Mathematics*, Oxford University Press, Oxford, 367-392.

[8] Carnap, R. (1931) Overcoming Metaphysics by Logical Analysis of Language [Überwindung der Metaphysik durch logische Analyse der Sprache]. *Erkenntnis*, **2**, 219-241. https://doi.org/10.1007/BF02028153

[9] Carnap, R. (1935) The Rejection of Metaphysics. In: Weitz, M., Ed., 20*th-Century Philosophy*: *The Analytic Tradition*, Free Press, New York, 206-220.

[10] Carnap, R. (1956) Meaning and Necessity: A Study in Semantics and Modal Logic. University of Chicago Press, Chicago and London.

[11] Carnap, R. (1966) Philosophical Foundations of Physics: An Introduction to the Philosophy of Science. Basic Books, New York.

[12] Carnap, R. (1967) The Logical Structure of the World. Pseudo-Problems in Philosophy. University of California Press, Berkeley and Los Angeles.

[13] Mach, E. (1914) The Analysis of Sensations, and the Relation of the Physical to the Psychical. Court Publishing Co., Chicago, London.

[14] Mach, E. (1919) The Science of Mechanics. A Critical and Historical Account of Its Development. Open Court Publishing Co., Chicago, London.

[15] Mach, E. (2006) Measurement and Representation of Sensations. Lawrence Erlbaum Associates, Inc., Mahwah.

[16] Neurath, O. (1983) Pseudorationalism of Falsification. In: Cohen, R.S., Neurath, M. and Neurath, O., Eds., *Philosophical Papers* 1913-1946, D. Reidel, Dordrecht, 121-131. https://doi.org/10.1007/978-94-009-6995-7_10

[17] Reichenbach, H. (1951) The Rise of Scientific Philosophy. University of California Press, Berkeley and Los Angeles. https://doi.org/10.1525/9780520341760

[18] Reichenbach, H. (1957) The Philosophy of Space and Time. Dover Publications, New York.

[19] Reichenbach, H. (1965) The Theory of Relativity and a Priori Knowledge. University of California Press, Berkeley.

[20] Reichenbach, H. (1969) Axiomatization of the Theory of Relativity. University of California Press, Berkeley.

[21] Reinchenbach, H. (1978) The Theory of Motion According to Newton, Leibniz, and Huyghens. In: Reichenbach, M. and Cohen, R.S., Eds., *Hans Reichenbach Selected Writings* 1909-1953, Vienna Circle Collection Book Series (VICC, Volume 4b), Springer, Dordrecht, 48-68. https://doi.org/10.1007/978-94-009-9855-1_2

[22] Reichenbach, H. (1980) From Copernicus to Einstein. Dover Publications, New York.

[23] Russell, B. (1914) Our Knowledge of the External World as a Field for Scientific Method in Philosophy [The Lowell Lectures, 1914]. Open Court Publ. Co., London.

[24] Russell, B. (1940) An Inquiry into Meaning and Truth. George Allen and Unwin, London.

[25] Russell, B. (1948) Human Knowledge: Its Scope and Limits. George Allen and Unwin, London.

[26] Russell, B. (1956) Logic and Knowledge. George Allen and Unwin, London.

[27] Russell, B. (1986) Is There an Absolute Good? *Russel*: *The Journal of Bertrand Russell Studies*, **6**, 144-149. https://doi.org/10.15173/russell.v6i2.1679

[28] Schlick, M. (1974) General Theory of Knowledge. Springer-Verlag, Wien. https://doi.org/10.1007/978-3-7091-3099-5

[29] Schlick, M. (1979) Philosophical Papers (Volume I). D. Reidel, Dordrecht.

[30] Schlick, M. (1979) Philosophical Papers (Volume II). D. Reidel, Dordrecht.

[31] Wittgenstein, L. (1992) Tractatus Logico-Philosophicus. Routledge & K. Paul, London, New York.

[32] Newton, I (1994) Mathematical Principles of Natural Philosophy. In: Adler, M.J., Ed., *Great Books of the Western World*, *Vol*. 32: *Newton*. *Huygens*, Encyclopedia Britannica, Inc., Chicago, London, 1-372.

[33] Newton, I. (2004) Newton: Philosophical Writings. Cambridge University Press, Cambridge. https://doi.org/10.1017/CBO9780511809293

[34] Kant, I. (1994) The Critique of Pure Reason. Fundamental Principles of the Metaphysics of Morals. The Critique of Practical Reason. Preface and Introduction to the Metaphysical Elements of Ethics. General Introduction to the Metaphysics of Morals. The Science of Right. The Critique of Judgement. In: Adler, M.J., Ed., *Great Books of the Western World*, *V*. 39: *Kant*, Encyclopedia Britannica, Inc., Chicago, London, 1-613.

[35] Kant, I. (1996) Prolegomena to Any Future Metaphysics: in Focus. Routledge, London, New York. https://doi.org/10.1017/CBO9781139164061

[36] Kant, I. (2004) Metaphysical Foundations of Natural Science. Cambridge University Press, Cambridge, New York. https://doi.org/10.1017/CBO9780511809613

[37] Schopenhauer, A. (2000) New Paralipomena [Novye paralipomeny]. In: Schopenhauer, A., Ed., *Introduction to Philosophy*. *New Paralipomena*. *Of Interesting*: *A Collection of Writings* [*Vvedenie v filosofiy*u. *Novye paralipomeny*. *Ob interesnom*: *Sbornik*], Popurri, Minsk, 55-389. (In Russian)

[38] Lobovikov, V.O. (2020) Knowledge Logic and Algebra of Formal Axiology: A Formal Axiomatic Epistemology Theory Sigma Used for Precise Defining the Exotic Condition under which Hume-and-Moore Doctrine of Logically Unbridgeable Gap between Statements of Being and Statements of Value Is Falsified. *Antinomies*, **4**, 7-23.

[39] Lobovikov, V.O. (2021) A Logically Formalized Axiomatic Epistemology System Σ + C and Philosophical Grounding Mathematics as a Self-Sufficing System. *Mathematics*, **9**, 1859. https://doi.org/10.3390/math9161859

[40] Lobovikov, V.O. (2021) A Formal Deductive Inference of the Law of Inertia in a Logically Formalized Axiomatic Epistemology System Sigma from the Assumption of Knowledge A-Priori-Ness. *Journal of Applied Mathematics and Physics*, **9**, 441-467. https://doi.org/10.4236/jamp.2021.93031

[41] Lobovikov, V.O. (2021) Formal Inferring the Law of Conservation of Energy from Assuming A-Priori-Ness of Knowledge in a Formal Axiomatic Epistemology System Sigma. *Journal of Applied Mathematics and Physics*, **9**, 1011-1040. https://doi.org/10.4236/jamp.2021.95070

[42] Lobovikov, V.O. (2022) Formally Deriving the Third Newton's Law from a Pair of Nontrivial Assumptions in a Formal Axiomatic Theory "Sigma-V". *Journal of Applied Mathematics and Physics*, **10**, 1561-1586. https://doi.org/10.4236/jamp.2022.105109

[43] Galilei, G. (1967) Dialogue Concerning the Two Chief World Systems—Ptolemaic and Copernican. University of California Press, Berkeley.

https://doi.org/10.1525/9780520342941

[44] Galilei, G. (1994) Dialogues Concerning the Two Sciences. In: Adler, M.J., Ed., *Great Books of the Western World*, *Vol*. 26: *Gilbert*. *Galileo*. *Harvey*, Encyclopedia Britannica, Inc., Chicago, London, 129-260.

[45] Drake, S. (1978) Galileo at Work: His Scientific Biography. University of Chicago Press, Chicago.

[46] Lobovikov, V.O. (2017) A Vector-Definition of Implication and A Vector-Definition of the Notion of "Law of Contraposition of Binary Operation" (A Structural-Functional Analogy between Logic and Pure-A-Priori Knowledge of Nature Exemplified by the Principle of Relativity of Velocity of Movement Discovered by Galileo Galilei) [Vektornoe opredelenie implikacii i vektornaya definiciya ponyatiya zakon kontrapozicii binarnoj operacii (Strukturno-funkcional'naya analogiya mezhdu logikoj i chistym estestvoznaniem a priori na primere otkrytogo Galileo Galileem principa otnositel'nosti skorosti dvizheniya)]. *Discourse-P*, **26**, 43-60. (In Russian) https://doi.org/10.17506/dipi.2017.26.1.4360

[47] Lobovikov, V.O. (2007) Mathematical Ethics, Metaphysics, and the Natural Law (Algebra of Metaphysics as Algebra of Formal Axiology) [Matematicheskaya etika, metafizika i estestvennoe pravo (Algebra metafiziki kak algebra formal'noi aksiologii)]. Institute of Philosophy and Law of the Ural Branch of Russian Academy of Sciences [Institut filosofii i prava Ural'skogo otdeleniya RAN], Yekaterinburg. (In Russian)

[48] Lobovikov, V.O. (2021) New Ideas in Philosophical Ontology and a Dual Code for Representing It in Artificial Intelligence Systems. Discrete Mathematical Modelling the Theory of Relativity of Space and Time by a System of Equations of Two-Valued Algebra of Formal Axiology [Novye idei v filosofskoj ontologii i dvoichnyj kod dlya ee predstavleniya v iskusstvennyh intellektual'nyh sistemah. Diskretnoe matematicheskoe modelirovanie teorii otnositel'nosti prostranstva i vremeni sistemoj uravnenij dvuznachnoj algebry formal'noj aksiologii]. *New Ideas in Philosophy* [*Novye idei v filosofii*], **8**, 11-20. (In Russian)

[49] Lobovikov, V.O. (2020) New Ideas in Philosophy of Dialectical Materialism: Justifying an Equivalence of "Being" and "Inner Contradictoriness of Matter" by "Computing" Corresponding Evaluation-Functions in Algebra of Formal Axiology [Novye idei v filosofii dialekticheskogo materializma: Obosnovanie ekvivalentnosti «bytiya» i «vnutrennej protivorechivosti materii» putem «vychisleniya» sootvetstvuyushchih cennostnyh funkcij v algebre formal'noj aksiologii]. *New Ideas in Philosophy* [*Novye idei v filosofii*], **7**, 17-28. (In Russian)

[50] Lobovikov, V.O. (2019) A Mathematical Model of "Dialectical Logic", and Relatively Autonomous Cognitive Robots [Matematicheskaya model' «dialekticheskoj logiki» i otnositel'no avtonomnye poznayushchie roboty]. *Antinomies* [*Antinomii*], **19**, 29-48. (In Russian) https://doi.org/10.17506/aipl.2019.19.1.2948

[51] Lobovikov, V.O. (2018) A Discrete Mathematical Model of the Dialectical Principle of Universal Interconnection [Diskretnaya matematicheskaya model' dialekticheskogo principa vseobshchej vzaimosvyazi]. *Discourse-P* [*Diskurs-Pi*], **30**, 118-128. (In Russian) https://doi.org/10.17506/dipi.2018.30.1.118128

[52] Corry, L. (2004) David Hilbert and the Axiomatization of Physics (1898-1918): From Grundlagen der Geometrie to Grundlagen der Physik (Archimedes: New Studies in the History and Philosophy of Science and Technology, Vol. 10). Springer Science + Business Media, B.V., Dordrecht, 530 p.

[53] Quinn, F. (2012) A Revolution in Mathematics? What Really Happened a Century

Ago, and Why It Matters Today. *Notices of the American Mathematical Society*, **1**, 1-11. https://doi.org/10.1090/noti858

[54] Tarski, A., Henkin, L. and Suppes, P. (1959) The Axiomatic Method with Special Reference to Geometry and Physics. North-Holland Publ. Co., Amsterdam.

[55] Lobovikov, V.O. (2018) Proofs of Logic Consistency of a Formal Axiomatic Epistemology Theory Ξ, and Demonstrations of Improvability of the Formulae $(Kq \to q)$ and $(\Box q \to q)$ in It. *Journal of Applied Mathematics and Computation*, **2**, 483-495. https://doi.org/10.26855/jamc.2018.10.004

[56] Ivin, A.A. (1970) Foundations of Evaluation Logic [Osnovanija logiki ocenok]. Izdatel'stvo Moskovskogo universiteta, Moscow. (In Russian)

[57] Mendelson, E. (1966) Introduction to Mathematical Logic. Second (Corrected) Edition, D. van Nostrand Co., Inc., Princeton.

[58] Hume, D. (1874) A Treatise of Human Nature Being an Attempt to Introduce the Experimental Method of Reasoning into Moral Subjects. In: Green, T.H. and Grose, T.H., Eds., *The Philosophical Works of David Hume in Four Volumes*, Vol. 2, Longmans, Green, and Co., London, 1-374.

[59] Moore, G.E. (2004) Principia Ethica. Dover Publications, Mineola.

[60] Guthrie, W.K.C. (1962) A History of Greek Philosophy. Vol. I: The Earlier Pre-Socratics and the Pythagoreans. The University Press, Cambridge.

[61] Makovel'skij, A. (1999) Pre-Socratics. Pre-Eleatic and Eleatic periods [Makovel'skij Dosokratiki. Doeleatovskij i eleatovskij periody]. Harvest, Minsk. (In Russian)

[62] Aristotle (1994) The Works of Aristotle. Vol. I. In: Adler, M.J., Ed., *Great Books of the Western World*, Vol. 7, Encyclopedia Britannica, Inc., Chicago, 1-726.

[63] Hume, D. (1994) An Enquiry Concerning Human Understanding. In: Adler, M.J., Ed., *Great Books of the Western World*, V. 33, Encyclopedia Britannica, Inc., Chicago, 451-509.

[64] Hume, D. (1998) An Enquiry Concerning the Principles of Morals. Oxford University Press, Oxford.

[65] Einstein, A., Lorentz, H.A., Minkowski, H. and Weyl, H. (1952) The Principle of Relativity. W. Perrett and G.B. Jeffery, Trans., Dover Books, New York.

[66] Einstein, A. (1994) Relativity: The Special and the General Theory. In: Adler, M.J., Ed., *Great Books of the Western World*, *Vol.* 56: 20*th Century Natural Science*, Encyclopedia Britannica, Inc., Chicago, 191-243.

[67] Guthrie, W.K.C. (1965) A History of Greek Philosophy. Vol. II: The Presocratic Tradition from Parmenides to Democritus. Cambridge University Press, Cambridge.

[68] Plato (1994) The Dialogues of Plato. In: Adler, M.J., Ed., *Great Books of the Western World*, *Vol.* 6: *Plato*, Encyclopedia Britannica, Chicago, 1-799.

[69] Plotinus (1991) The Enneads. Translated by Stephen MacKenna, Abridged and Edited by John Dillon, Penguin Books, London.

[70] Guthrie, W.K.C. (1981) A History of Greek Philosophy. Vol. VI: Aristotle an Encounter. The University Press, Cambridge.

[71] Guthrie, W.K.C. (1975) A History of Greek Philosophy. Vol. IV: Plato: The Man and His Dialogues: Earlier Period. The University Press, Cambridge.

[72] Guthrie, W.K.C. (1978) A History of Greek Philosophy. Vol. V: The Later Plato and the Academy. The University Press, Cambridge.

[73] Leibniz, G.W. (1982) Essays in Four Volumes. V. 1 [Sochineniya v chetyrekh to-

mah. T. 1]. Mysl', Moscow. (In Russian)

[74] Lobovikov, V.O. (2021) A Logically Formalized Axiomatic Epistemology System Ksi Modeling Kant's Extraordinary Statement of Physicist's Prescribing A-Priori Laws to Nature. *Discourse-P* [*Diskurs-Pi*], **18**, 142-157. https://doi.org/10.17506/18179568_2021_18_2_142