УДК 1(091)+16+17+51-7+510.2+510.687+510.82+512
DOI 10.47850/S.2022.1.7

AXIOMATIZING PHILOSOPHICAL EPISTEMOLOGY, A FORMAL THEORY "SIGMA + 2C" AND PHILOSOPHICAL FOUNDATIONS OF MATHEMATICS

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The paper is devoted to investigating Kant's apriorism underlying Hilbert's formalism in philosophical foundations of mathematics. The target is constructing a formal axiomatic theory of knowledge in which it is possible to invent formal inferences of formulae-modeling-Hilbert-formalism from the assumption of Kant apriorism concerning mathematics. The s*cientific novelty*: a logically-formalized axiomatic system of universal philosophical epistemology called "Sigma +2C" is invented for the first time as a generalization of the already published formal epistemology system "Sigma +C". In comparison with "Sigma +C", a new symbol is included into the object-language-alphabet of Σ+2C, namely, the symbol standing for the *perfection*-modality "it is *complete* that…". Also, one of axiom-schemes of "Sigma +C" is generalized in "Sigma + 2C". In "Sigma +2C", it is proved deductively that under the assumption of *a-piori*-ness of mathematical knowledge, its completeness and consistency are equivalent.

Keywords: formal axiomatic theory of knowledge; a-priori knowledge; empirical knowledge; Kant's apriorism; Hilbert's formalism; Gödel's incompleteness theorem; two-valued algebraic system of formal axiology.

АКСИОМАТИЗАЦИЯ ФИЛОСОФСКОЙ ЭПИСТЕМОЛОГИИ, ФОРМАЛЬНАЯ ТЕОРИЯ «СИГМА + 2С» И ФИЛОСОФСКИЕ ОСНОВАНИЯ МАТЕМАТИКИ

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Доклад посвящен исследованию кантовского априоризма, являющегося предпосылкой формализма Гильберта в философских основаниях математики. Цель – построение некой формальной аксиоматической теории знания, в которой возможно построение формальных выводов формул, моделирующих формализм Гильберта, из допущения кантовского априоризма математического знания. *Научная новизна*: впервые построена некая логически формализованная аксиоматическая система универсальной философской эпистемологии «Сигма + 2С» как обобщение уже опубликованной системы формальной эпистемологии «Сигма + С». В сравнении с «Сигма + С», некий новый символ включен в алфавит языка-объекта «Сигма + 2С», а именно, символ, обозначающий *модальность идеала (совершенства)* «это *полно*, что …». Также, в системе «Сигма + 2С», одна из схем аксиом системы «Сигмы + С» существенно обобщена. В «Сигма + 2С» дедуктивно доказано, что при допущении априорности математического знания, его полнота и непротиворечивость эквивалентны.

Ключевые слова: формальная аксиоматическая теория знания; априорное знание; эмпирическое знание; априоризм Канта; формализм Гильберта; теорема Гёделя о неполноте; двузначная алгебраическая система формальной аксиологии.

1. Introduction

There are infinitely many different modal logics. The number of possible combinations of different kinds of modalities is immense. Even within the scope of modal logic of knowledge we need a set of *significantly different* modalities called "knowledge", various combinations of which make different *multimodal* epistemic logics. As in the intellectually respectable definitions of the notion "knowledge" the words "true" and "provable" are exploited necessarily, the modal logic treating *truth as modality* and the modal logic treating *provability as modality* are indispensable for epistemology. In content philosophy the word-homonym "knowledge" is naturally connected with many other modal terms (alethic, deontic, axiological, et al), consequently, while inventing and elaborating a hypothetical multimodal formal axiomatic system of universal philosophical epistemology, one has to utilize not only proper-epistemic modalities but also many other concepts of modal metaphysics. This is just what I am to do in the present article, namely, I am to invent (construct) a novel logically formalized axiomatic system of *multimodal* philosophy of knowledge. However, the concrete theme and the goal of the paper necessitate a restriction of the set of different kinds of modalities to be involved into the discourse.

The list of various kinds of modalities to be taken into an account in this article is determined by the subject-matter and target of the research. At present moment I am equipped with the axiomatic epistemology systems Σ and Σ+C, which are already published in [Lobovikov, 2020; 2021], respectively. However, I think that for realizing the goal of the paper, Σ is not quite sufficient, and Σ+C is not optimal. For the optimization, it is worth adding to Σ not only the *modality of consistency* (the first "C", which has been added to Σ in Σ+C), but also the *modality of completeness* (the second "C", which is to be added to Σ+C in Σ+2C submitted below in this article).

In this paper, the above-mentioned significantly *novel mutation* of the logically formalized *multimodal* axiomatic epistemology system Σ is to be used for logical analyzing a system of philosophical foundations of mathematics which (system) is made up by the following set of statements ST1 – ST8:

ST1: proper mathematical knowledge of ω is *a-priori* one. See, for instance, [Kant, 1994, p. 16, 18].

ST2: truth of ω and provability of ω are logically equivalent in the rationalistic optimism *ideal* created by G. W. Leibniz [1903; 1969; 1981] and D. Hilbert [1990; 1996a–1996c]. Of the rationalistic optimism *ideal* and K. Gödel's philosophy see also the article by V. V. Tselishchev [2013]. By the way, here it is relevant to note that there is a nontrivial *formal-axiological* equivalence of "true" and "provable" [Lobovikov, 2009] but the almost unknown "*formal-axiological* equivalence" and the well-known "formal-logical one" are not identical.

ST3: consistency of proper mathematical statement or theory ω and provability of consistency of ω are logically equivalent in Hilbert's *ideal* of self-sufficient (self-dependent) mathematics;

ST4: truth of ω and consistency of ω are logically equivalent (in the ideal).

ST5: truth of ω is logically equivalent to ω (in the ideal).

ST6: consistency of ω is logically equivalent to completeness of ω (in the ideal).

ST7: truth of ω and completeness of ω are logically equivalent (in the ideal).

ST8: completeness of proper mathematical theory (or statement) ω and provability of completeness of ω in a consistent theory are logically equivalent in the *ideal* of self-sufficient (self-dependent) mathematics.

D. Hilbert was not alone; his rationalistic optimism *ideal* (norm) of mathematical activity was attractive also for A. Tarski and for many other prominent mathematicians. Even being aware of Gödel's theorems of incompleteness, A. Tarski believed and wrote that *it is good (desirable)* for a mathematician to prove that ST2 is true in relation to a concrete mathematical statement or theory ω, if this proving is possible [1948, pp. 185–189]. Also being aware of Gödel's theorems of incompleteness and taking them into an account, V. V. Tselishchev writes (in perfect accordance with Tarski) that proving consistency and completeness is a *norm (duty)* which is *prescribed (obligatory)* for a mathematician, if such proving is possible [Tselishchev, 2004a; 2004b; 2005]. Here the famous bimodal Kant-principle "obligation (duty) implies possibility" $(Op \supset \Diamond p)$ works. As due to the theorems by Gödel, proving completeness of the formal arithmetic system (under the condition of its consistency) is impossible, there is no violation of the *norm* (the relevant *obligation* is abolished by *modus tollens*).

If Hilbert's formalism ideal and program of/for philosophical grounding mathematics was fulfilled (i.e. if the ideal created by him was realized), then the system of mathematical knowledge (as a whole) would be *self-sufficient (self-dependent)* one. Unfortunately, today there is a widespread opinion (a *statistical norm* of thinking and affirming) that Hilbert's ideal and the formalism program targeted at realizing this ideal were totally annihilated by Gödel's theorems of incompleteness. However, the widespread opinion is not able to explain the reason (philosophical foundation) of/for Hilbert's creating the ideal and the formalism program in question. The folks talking of Gödel's termination of Hilbert's formalism program do not recognize a possibility of existence of a *not empty* domain in which Hilbert's ideal and the formalism program targeted at realizing this ideal are perfectly adequate even today (and forever). If so, then significance of Gödel's famous results is reduced to significance of precise limiting the mentioned *not empty* domain, i.e. to significance of establishing quite exact border-lines of/for that domain. The present paper is aimed at recognizing and explicating the strong *reason* of/for Hilbert's creating the formalism program and at giving an exact definition of the realm of the program's soundness missed by the mentioned folks.

By analyzing the above statements ST1 – ST8, it is possible to focus on the set of *qualitatively different* modalities which are indispensable for formulating ST1 – ST8, namely the following: "*knows* that…"; "*a-priori knows* that…"; "*empirically* *knows* that…"; "*it is true* that…"; "it is *provable* in a consistent theory that…", "it is *consistent* that…", "it is *complete* that…". The first five modalities are taken into an account by Σ while the last two ones are not. Therefore, successfully to cope with realizing the research goal, it is worth making a mutation in Σ+C by adding the novel modality "*Completeness*" to it. In Σ+C, the symbol Cω stands for "it is *consistent* that ω". As now the novel modality "Completeness" is added to Σ+C, let the symbol "Σ+2C" be the name of/for the result of adding the two modalities (Consistency and Completeness) to Σ. Thus, the general idea of this article is introduced in first approximation which is sufficient to begin with. Now let us move to the next paragraph giving a precise definition of the multimodal formal axiomatic system Σ+2C to be used in this paper as an effective means of/for realizing the goal.

2. A New Formal Multimodal Axiomatic Epistemology-and-Axiology Theory Σ+2C

In result of (1) adding the modality C ("It is *consistent* that…") to the set of *perfection*-modalities of the multimodal system Σ, and (2) significant *generalizing* the axiom-scheme AX-5 of Σ, a new system (named "Σ+C") has come into being. The axiomatic system Σ+2C is a result of developing further the formal axiomatic *epistemology* theory Σ [Lobovikov, 2020] and the formal axiomatic *epistemology-and-axiology* theory Σ+C [Lobovikov, 2021]. As I have to minimize number of repetitions of sentences and phrases already used by me in my own texts already published somewhere, I have to abstain from manifestly giving precise definitions of the notions: "alphabet of object-language of Σ+2C", "term of Σ+2C", "formula of Σ+2C", and some other. Definitions of these notions of Σ+2C look similar to the corresponding definitions of notions of Σ. The reader can find the definitions of relevant notions of Σ in the already published (open access) articles [Lobovikov, 2020; 2021].

Proper logic axioms and inference rules of Σ, Σ+C, and Σ+2C are the ones of classical logic of propositions. Thus, the proper logic foundations of Σ, Σ+C, and Σ+2C are identical but the logically formalized systems constructed on these foundations are different. Although, at first glance, corresponding definitions of Σ, Σ+C, and Σ+2C seem identical, strictly speaking, they are not identical. The theories Σ, Σ+C, and Σ+2C have different alphabets of their object-languages, different sets of expressions, different sets of formulae, different sets of axioms, different sets of theorems.

The modality symbols exploited in the present article are introduced as follows. Symbols K, A, E, S, T, F, P, D, respectively, stand for modalities "agent *Knows* that…", "agent *A-priori knows* that…", "agent *Empirically* (*a-posteriori) knows* that…", "under some conditions in some space-and-time a person (immediately or by means of some tools) *Sensually perceives* (has *Sensual verification*) that…", "it is *True* that…", "person has *Faith* (or believes) that…", "it is *Provable* in a consistent theory that…", "there is *an algorithm* (a machine could be constructed) *for Deciding* that…".

Symbols C, Y, G, W, O, B, U, J, respectively, stand for modalities "it is *Consistent* that…", "it is *Complete* that…", "it is *(morally) Good* that…", "it is *(morally) Wicked* that…", "it is *Obligatory* that …", "it is *Beautiful* that …", "it is *Useful* that …", "it *is Joyful, pleasant* that…".

In this paragraph, syntax meanings of the modality symbols are defined precisely (although not manifestly) by the following schemes of proper axioms of multimodal philosophy theory Σ+2C.

AX-1: $A\alpha \supset (\Box\beta \supset \beta)$.

AX-2: $A\alpha \supset (\Box(\alpha \supset \beta) \supset (\Box\alpha \supset \Box\beta))$.

AX-3: $A\alpha \leftrightarrow (K\alpha \;\&\; (\neg\Diamond\neg\alpha \;\&\; \neg\Diamond S\alpha \;\&\; \Box(\beta \leftrightarrow \Omega\beta)))$.

AX-4: $E\alpha \leftrightarrow (K\alpha \;\&\; (\Diamond\neg\alpha \vee \Diamond S\alpha \vee \neg\Box(\beta \leftrightarrow \Omega\beta)))$.

AX-5: $\Omega\alpha \supset \Diamond\alpha$.

AX-6: $(\Box\beta \;\&\; \Box\Omega\beta) \supset \beta$.

AX-7: $(t_i{=}{+}{=}t_k) \leftrightarrow (G[t_i] \leftrightarrow G[t_k])$.

AX-8: $(t_i{=}{+}{=}g) \supset \Box G[t_i]$.

AX-9: $(t_i{=}{+}{=}b) \supset \Box W[t_i]$.

AX-10: $(G\alpha \supset \neg W\alpha)$.

AX-11: $(W\alpha \supset \neg G\alpha)$.

Definition scheme DF-1: $\Diamond\gamma$ is a *name* of/for $\neg\Box\neg\gamma$ (where γ is a formula of Σ+2C).

In AX-3, AX-4, AX-5, and AX-6, the symbol Ω (belonging to the metalanguage) stands only for a (any) "perfection-modality". Not all the above-mentioned modalities are perfection-ones. The set Δ of "perfections" (perfection-modalities) is the following {K, D, F, C, Y, P, J, T, B, G, U, O, $\Box$}.

Evidently, Δ is a subset of the set of all the modalities under consideration in this article. Including C and Y into the set Δ of *perfection*-modalities is quite natural as "consistency" and "completeness" are important *perfections* of a theory (Tarski, p. 185, 186). As a rule, *de-dicto*-modalities are attached to a *dictum*. Usually, the word "*dictum"* is translated (interpreted) from the Latin language as a "*proposition* (or sentence)", but, in principle, it is possible to generalize the meaning of the word "*dictum*" in such a way that a theoretical (deductive) system would be a *dictum* as well.

A justification of AX-10 and AX-11 can be found in formal logic of evaluations and preferences. But the almost unknown (aunhabitual) axiom-schemes AX-7, AX-8, and AX-9 represent not the formal logic but a formal axiology (universal theory of value forms). The notion "formal logic" is not logically equivalent to the notion "formal axiology", consequently, "formal-logic inconsistency" and "formal-axiological one" are not synonyms. The significant logic-difference between notions "*formal-axiological* contradiction" and "formal-logic one" explains a psychologically unexpected possibility of deductive proof of the *formal-axiological inconsistency* of the formal arithmetic theory [Lobovikov, 2011a; 2011b].

3. Defining semantics of the theory Σ+2C

In the above section 2, the definition of Σ+2C has been deliberately deprived of its philosophical contents (owing to the relevant abstraction). The axiomatic theory Σ+2C is a *multimodal* one, but hitherto in this paper concrete contents of modalities have been exposed not sufficiently; the theory Σ+2C has been considered as actually formal one. Below in this section we are to relax the formality of Σ+2C and to move to concrete contents of the modalities under consideration in Σ+2C. On the syntax level of Σ+2C, meanings of the modal symbols are defined by the above-given axiom-schemes.

In this article it is presumed that semantic meanings of the proper logic symbols of the artificial language of classical logic of propositions are well-defined by relevant handbooks, hence, there is no need to define them here. On the contrary, the extraordinary (very unusual) signs of the artificial language of Σ+2C require a systematical specification of their semantic meanings.

Defining semantic meanings is defining an interpretation-function. To define the interpretation-function one has to define (1) a set which plays the role of "domain (or field) of interpretation" (let the interpretation-domain be denoted by the letter M) and (2) a "valuator (evaluator)" *V*. By definition, in a standard interpretation of Σ+2C, M is such a set, every element of which has: (1) one and only one *axiological value* from the set {good, bad}; (2) one and only one *ontological value* from the set {exists, not-exists}.

The *axiological variables* (z, x, y, z_i, x_k, y_m) take their values from the set M.

The *axiological constants* "b" and "g" mean "bad" and "good", respectively.

Valuating an element from M by a concrete (fixed) interpreter V is ascribing an *axiological value* (either good or bad) to that element. The interpreter V may be either collective or individual one. Certainly, a change of V can change some relative evaluations, but cannot change the set of laws of two-valued algebra of formal axiology which are not relative but absolute evaluations, namely, such and only such *constant valuation-functions* which have the value g (good) under any possible combination of axiological values of their axiological variables. Although V is a variable taking its values from the set of all possible interpreters, a perfectly defined interpretation of Σ+2C necessarily implies that the value of V is fixed. A change of V necessarily implies a change of interpretation.

In the present article, "e" and "n" stand for "… exists" and "… does-not-exist", respectively. The signs "e" and "n" are named "*ontological constants*". By definition, in a standard interpretation of Σ+C, one and only one element of the set {{g, e}, {g, n}, {b, e}, {b, n}} corresponds to every element of M. The signs "e" and "n" belong to the meta-language. By definition of the alphabet of object-language of Σ+C, "e" and "n" do not belong to the object-language. Nevertheless, "e" and "n" are *indirectly* represented at the level of object-language of Σ+2C by means of *square-bracketing*: "t_i exists" is represented by $[t_i]$; "t_i does not exist" is represented by $\neg[t_i]$. This means that square-bracketing is a significant part of exact defining formal-axiological-and-ontological semantics of Σ+2C.

N-placed terms of Σ+2C are interpreted as *n*-placed evaluation-functions defined on the set M. "One-placed evaluation-function" is exemplified below by the **Table 1**. (It is relevant to recall here that the upper index 1 standing immediately after a capital letter means that this letter stands for a one-placed evaluation-function.)

Table 1. Definition of the functions determined by one evaluation-argument

x	B_1^1x	N_1^1x	C_1^1x	I_1^1x	Z_1^1x	S_1^1x	U_1^1x	A_1^1x	G_1^1x	P_1^1x	H_1^1x	R_1^1x
g	g	b	g	b	b	b	b	g	g	g	b	b
b	b	g	b	g	b	b	b	g	g	b	g	g

In the **Table 1**, the one-placed term B_1^1x is interpreted as one-placed evaluation-function "being (existence) of (what, whom) x"; the term N_1^1x is interpreted as evaluation-function "non-being (nonexistence) of (what, whom) x". C_1^1x – "consistency of (what, whom) x". I_1^1x – "inconsistency of (what, whom) x". Z_1^1x – "formal-axiological inconsistency (or absolute inconsistency) of (what, whom) x". S_1^1x – "x's self-contradiction" U_1^1x – "absolute non-being of (what, whom) x". A_1^1x – "absolute being of (what, whom) x". G_1^1x – "absolute goodness of (what, whom) x", or "absolute good (what, who) x". P_1^1x – "positive evaluation of (what, whom) x". H_1^1x – "negative evaluation of (what, whom) x". R_1^1x – "resistance to (what, whom) x".

The notion "*two-placed evaluation-function*" is instantiated by the below **Table 2**. (In this paper, the upper index 2 standing immediately after a capital letter means that this letter stands for a two-placed function.)

Table 2. Definition of the evaluation-functions determined by two arguments

x	*y*	K^2xy	S^2xy	X^2xy	T^2xy	Z^2xy	P^2xy	C^2xy	E^2xy	V^2xy	N^2xy	Y^2xy
g	g	g	b	b	b	b	g	g	g	b	b	g
g	b	b	g	b	b	b	g	b	b	g	b	g
b	g	b	g	g	g	g	b	g	b	g	b	g
b	b	b	g	b	b	b	g	g	g	b	g	b

In the **Table 2**, the two-placed term K^2xy is interpreted as evaluation-function "*being of both x* and *y* together", or "*joint being* of *x* with *y*". S^2xy is interpreted as "*separation, divorcement* between *x* and *y*. The term X^2xy – evaluation-function "*y's being without x*", or "joint being of *y* with nonbeing of *x*". T^2xy – "*termination* of *x* by *y*". Z^2xy – "*y*'s *contradiction* to (with) *x*". P^2xy – "*preservation, conserva-*

tion, protection of x by y". C^2xy is interpreted as evaluation-function "*y's existence, presence in x*". E^2xy – "*equivalence, identity (of values)* of x and y". V^2xy – "*choosing and realizing* such and *only such* an element of the set $\{x, y\}$, which is: 1) *the best* one, if both x and y are good; 2) the least bad one, if both x and y are bad; 3) the good one, if x and y have opposite values. (Thus, V^2xy means an *excluding choice and realization of only the optimal* between x and y.) The term N^2xy is interpreted as evaluation-function "*realizing neither x nor y*". Y^2xy is interpreted as evaluation-function "realizing a *not-excluding-choice* result, i.e. 1) *realizing* K^2xy *if both x and y are good*, and 2) *realizing* V^2xy *otherwise*".

To exclude possibilities of misunderstanding this paper, it is quite relevant to highlight that in a standard interpretation of Σ+2C, the signs B_1^1x, N_1^1x, C_1^1x, K^2xy, C^2xy, E^2xy, V^2xy stand not for predicates but for *n*-placed evaluation-functions. Being given an interpretation of Σ+2C, such expressions of the object-language of Σ+2C, which have forms $(t_i{=}{+}{=}t_k)$, $(t_i{=}{+}{=}g)$, $(t_i{=}{+}{=}b)$, are representations of *predicates* in Σ+2C.

By definition of semantics of Σ+2C, if t_i is a term of Σ+2C, then, being interpreted, a formula (of Σ+2C), which has the form $[t_i]$, is an *either true or false proposition* "t_i exists". Thus, by definition, in a standard interpretation, formula $[t_i]$ is true if and only if t_i has the *ontological value* "e (exists)" in that interpretation. Also, by definition, the formula $[t_i]$ is false in a standard interpretation of Σ+2C, if and only if t_i has the ontological value "n (does not-exist)" in that interpretation.

By definition of semantics of Σ+2C, in a standard interpretation of Σ+2C, the formula scheme $(t_i {=}{+}{=} t_k)$ is a proposition possessing the form "t_i is *formally-axiologically equivalent* to t_k"; this proposition is true if and only if (in that interpretation) the terms t_i and t_k obtain identical *axiological values (*from the set {good, bad}) under any possible combination of *axiological values* of their *axiological variables*.

By definition of semantics of Σ+2C, in a standard interpretation of Σ+2C, the formula scheme $(t_i {=}{+}{=} b)$ is a proposition having the form "t_i is a *formal-axiological contradiction*" (or "t_i is *formally-axiologically, or invariantly, or absolutely bad*"); this proposition is true if and only if (in that interpretation) the term t_i acquires axiological value "bad" under any possible combination of axiological values of the axiological variables.

By definition of semantics of Σ+C, in a standard interpretation of Σ+2C, the formula scheme $(t_i {=}{+}{=} g)$ is a proposition having the form "t_i is a *formal-axiological law*" (or "t_i is *formally-axiologically, or invariantly, or absolutely good"*); this proposition is true if and only if (in the interpretation) the term t_i acquires *axiological value* "good" under any possible combination of axiological values of the axiological variables.

In respect to the above-given definition of sematic meaning of $(t_i =+= t_k)$ in Σ+2C, it is indispensable to highlight the important linguistic fact of homonymy of the words “is”, “means”, “implies”, “entails”, “equivalence” in natural language. On one hand, in natural language, these words may have the well-known formal logic meanings. On the other hand, in natural language, the same words may stand for the above-defined *formal-axiological-equivalence* relation “=+=”. This ambiguity of natural lenguage is to be taken into an account; the different meanings of the homonyms are to be separated systematically; otherwise the homonymy can head to logic-linguistic illusions of paradoxes.

Owing to the above-presented definition of formal-axiological-and-ontological semantics of Σ+2C, it is easy to recognize that the two-valued algebraic system of formal axiology is nothing but abstract *theory-of-relativity* of evaluations; in this theory-of-relativity, the *formal-axiological laws* (constantly good evaluation-functions) of that algebraic system are *invariants* in relation to all possible transformations of interpreter V. Thus, although it is a perfectly evident fact that relativity (and mutability) of empirical valuations does exist, the valuation-invariants (immutable universal laws of valuation-relativity) do exist as well.

The above-submitted material is published to support understanding the presentation of the paper, in which the above-promised formal deductive proofs of the philosophically intereseting theorems are to be submitted. The deductive proofs are intentionaly omitted in this text due to its page limits, but the formal inferences and their discussions are to be performed during the paper presentation at the conference.

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