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THE LOGICAL SQUARE AND HEXAGON OF OPPOSITION AS GRAPHIC MODELS OF AN UNKNOWN FORMAL-AXIOLOGICAL INTERPRETATION OF LEIBNIZ' PRINCIPLE OF IDENTITY OF INDISCERNIBLES

Introduction

Today, plenty of interesting interpretations, explications and discussions of Leibniz's principle of identity of indiscernibles have been accomplished, for example.[1] In contemporary philosophical logic literature, the principle has been explicated by means of the symbolic logic of predicates[2]. However, as a rule, the explications and discussions are confined within the realm of *descriptive-indicative* meanings of the words: "identity", "difference", etc. Traditionally, a discussion of the principle deals with proper *ontology* statements (criterions) of being or nonbeing of identity or nonidentity. As a rule, proper *axiological (evaluative)* aspect of the principle is missed (omitted by negligence or ignored on principle). Perhaps, Leibniz and his followers had concentrated on investigating exactly *descriptive-indicative* meaning of the term "identity of indiscernibles", by accepting the scientific abstraction from *proper evaluative (axiological)* meaning of the term. In any way, there is a still not well-recognized possibility to create and elaborate systematically a hitherto unknown *formal-axiological* interpretation of Leibniz's principle of identity of indiscernibles, in addition to the well-studied *formal-logical* interpretations and explications of the principle. I believe that there is a fundamental *analogy* between abstract formal logic of thinking and abstract formal axiology of acting, hence, it is truthlike that along with the well-known logical square of opposition and with its generalizations by A. Sesmat, R. Blanché, J.-Y. Béziau, and others[3], there are unknown

1 P. Forrest: "The Identity of Indiscernibles", in: Edward N. Zalta (ed.): *The Stanford Encyclopedia of Philosophy* (Winter 2020 Edition), https://plato.stanford.edu/archives/win2020/entries/identity-indiscernible/; I. Hacking: "The Identity of Indiscernibles", in: *Journal of Philosophy* 72/9 (1975), pp. 249–256; K. Hawley: "Identity and Indiscernibility", in: *Mind* 118 (2009), pp. 101–9; A. Jauernig: "The Modal Strength of Leibniz's Principle of the Identity of Indiscernibles", in: *Oxford Studies in Early Modern Philosophy* 4 (2008), pp. 191–225; B.C. Look: "Gottfried Wilhelm Leibniz", in: Edward N. Zalta (ed.): *The Stanford Encyclopedia of Philosophy* (Spring 2020 Edition), https://plato.stanford.edu/archives/spr2020/entries/leibniz/; G. Rodriguez-Pereyra: "Leibniz's Argument for the Identity of Indiscernibles in His Correspondence with Clarke", in: *Australasian Journal of Philosophy* 77 (1999), pp. 429–38.

2 B. C. Look: "Gottfried Wilhelm Leibniz".

3 A. Sesmat: *Logic. I, II. Reasoning, Logistics* [Logique. I, II. Les raisonnments, la logistique], Paris 1950–51; R. Blanché: *Intellectual Structures [Structures intellectuelles]*, Paris 1966; J.-Y. Béziau: "The New Rising of the Square of Opposition", in: J.-Y. Béziau, D. Jacqette (eds.): *Around and Beyond the Square of Opposition*, Basel 2012, pp. 3–19; J.-Y. Béziau: "The Power of the Hexagon", in: *Logica Universalis* 6/1–2 (2012), pp. 1–43; A. Moretti: *The Geometry of*

formal-axiological squares, hexagons, octagons, n-gons, and other graphic models of *formal-axiological opposition of action forms* deprived of their concrete (moral, legal, aesthetic, etc.) *evaluative* contents. The system of rules of the here-submitted hypothetic *formal-axiological* square and hexagon is a system of *formal-axiological* rules of activity in general and of its proper metaphysical aspect especially. The hitherto unknown system of formal-axiological rules is *analogous* to the well-known system of formal-logic rules modelled by the geometric figures. However, the elegant *structural-functional analogy* is not an *equivalence* of the analogous rule systems, therefore, the formal-logical and formal-axiological interpretations of the geometric figures are not identical. Hence, the paper submits quite a new scientific result worthy of discussing and taking into an account. As the *formal-axiological* interpretation and investigation of metaphysics in general, and of Leibniz' principle of identity of indiscernibles in particular, is accomplished in this paper by means of the still-almost-unknown two-valued *algebraic system of metaphysics as formal axiology*[4], it is necessary to introduce and to define precisely the minimal set of basic notions of that algebraic system. An exact formulation of the system is placed immediately below.

A Precise Definition of Two-Valued Algebraic System of Metaphysics as Formal Axiology

The two-valued algebraic system of metaphysics as formal axiology is nothing but a triple < Φ, Ω, R > in which the sign Φ denotes the set of all such and only such *either-existing-or-not-existing units* which are *either good or bad* ones from the viewpoint of a *valuator* Σ. The sign Σ denotes a person (individual or collective one – it does not matter), in respect to which all assessments are performed. Certainly, Σ is a *variable*: changing values of Σ can result in changing assessments of concrete elements of Φ. However, if a value of the variable Σ is perfectly fixed, then assessments of concrete elements of Φ are quite definite (not relative). Elements of Φ are called formal-axiological-objects of metaphysics. The signs "*g*" (good), and "*b*" (bad) stand for *abstract axiological values* of elements of Φ. Moral actions or persons (individual or collective – it does not matter) are concrete instances (particular cases) of elements of Φ. In < Φ, Ω, R >, the sign Ω denotes the set of all *n-ary algebraic operations* defined on the set Φ. (These algebraic operations are called *formal-axiological* ones.) In the mentioned triple, the symbol R denotes the set of

Opposition. Ph.D Thesis, Neushâtel 2009; A. Moretti: "Why the Logical Hexagon?",in: *Logica Universalis* 6 (2012), pp. 69–107; W. Lenzen: "Leibniz' Logic and the 'Cube of Opposition'",in: *Logica Universalis* 10/2–3 (2016), pp. 171–189; D. Jaspers: "Logic and Color",in: *Logica Universalis* 6 (2012), pp. 227–248; L. Demey/H. Smessaert: "Metalogical Decorations of Logical Diagrams",in: *Logica Universalis* 10/2–3 (2016), pp. 233–292.

4 V. O. Lobovikov: "Algebra of formal axiology as a discrete mathematical model of philosophy",in: N.V. Bryanik (ed.): *At Philosophical Crossroads [Na filosofskih perekrestkah]*, Moscow/Yekaterinburg 2019, pp. 244–285 (in Russian).

all *n-ary formal-axiological relations* defined on the set Φ. (For instance, the below-defined binary relation "*formal-axiological equivalence*" belongs to R.)

Algebraic operations defined on the set Φ are *abstract-value-functions*. *Abstract-value-variables* of these functions take their values from the set {g (good), b (bad}. Here the signs "g" and "b" denote the abstract axiological values "good" and "bad", respectively. The functions take values from the same set.

In the talk of *abstract-value-functions*, the following mappings are meant: $\{g, b\} \rightarrow \{g, b\}$, if one talks of the functions determined by *one abstract-value-argument*; $\{g, b\}\times\{g, b\} \rightarrow \{g, b\}$, if one talks of the functions determined by *two abstract-value-arguments* (here "×" denotes the Cartesian product of sets); $\{g, b\}^N \rightarrow \{g, b\}$, if one talks of the functions determined by *N abstract-value-arguments*, (here *N* is a finite positive integer).

In algebra of formal axiology, the signs "*x*" and *"y*" denote *abstract-value-forms* of elements of Φ. (Moral-value-forms of actions and persons are *concrete instances* or particular cases of *abstract-value-forms* of elements of Φ.) Elementary abstract-value-forms deprived of their specific contents represent independent abstract-value-arguments. Complex abstract-value-forms deprived of their specific contents represent abstract-value-functions determined by these arguments. In this paper, only some abstract-evaluation-functions determined by *one* abstract-evaluation-argument are considered, namely, the functions defined below by Table 1, and Table 2.

Table 2. The evaluation-functions determined by one evaluation-argument

x	*P* *x*	*I* *x*	W x	*B* *x*	*N* *x*	*M* *x*	*D* *x*	D_1 *x*	*R* *x*	M_1 *x*	*E* *x*	*T* *x*	B_1 *x*	B_2 *x*	*U* *x*	M_2 x	D_3 *x*	*L* *x*
g	G	b	g	G	b	g	G	B	B	b	b	B	g	b	g	b	b	b
b	B	g	b	B	g	b	B	G	G	g	g	G	b	g	b	g	g	g

Glossary for Table 1. The symbol *Px* stands for the evaluation-function "*possibility of* (what, whom) *x*". The symbol *Ix* stands for the evaluation-function "*impossibility of x*". The sign *Wx* denotes the function "*universe of x*", or "*x*'s *world*". *Bx* denotes the evaluation-function "*being of x*". *Nx* denotes the evaluation-function "*nonbeing of x*". *Mx* stands for the function "*monad of x*". *Dx* – the function "*different, discernible* (what, who) *x*", or "*x*'s *being discernible, different*". D_1x – the function "*different, discernible from x*". *Rx* – "*relative* (what, who) *x*", or "*relativeness* of *x*". M_1x – "*matter*, *material*, *materialness of* (what, whom) *x*". *Ex* – "*external, outer* (what, who) *x*". *Tx* – "*transcending, crossing*, overcoming, going beyond, cutting across (what, whom) *x*". B_1x – "(own) *border of* (what, whom) *x*", or "*x*'s (own) border". B_2x – "*border for* (what, whom) *x*". *Ux* – "*unity, oneness of x*. M_2x – "*many-ness of x*". D_3x – "*division, divisibility, dividedness of x*". *Lx* – "limited, restricted, confined (what, who) *x*". These evaluation-functions are defined by Table 1.

Table 2. The one-placed evaluation-functions

x	Fx	I_1x	Gx	Ox	Vx	Sx	S_1x	D_4x	D_5x	T_1x	Ax		P_1x	F_1x	Cx	T_2x	C_1x	C_2x	S_2x
g	B	g	g	B	b	b	B	B	b	g	g		B	g	b	b	g	g	g
b	G	b	g	G	g	b	G	B	g	b	g		G	b	g	g	b	g	b

Glossary for Table 2. Fx – "*finiteness*, definiteness, localness, temporality of x", or "*finite*, definite, local, temporal x". I_1x – "*infiniteness*, indefiniteness, eternity of x", or "*infinite*, indefinite, eternal x". Gx – "*God* of (what, whom) x in a monotheistic world religion". Ox – "*opposite* of/for x". Vx – "*contradiction to* (with) x". Sx – "*self-contradiction* to (with) x". S_1x – "*inner contradictoriness of* x". D_4x – "*self-difference, self-differentiation of* x". D_5x – "*inner distinction of* x". T_1x – "*time of* x". Ax – "*absolute time of* x". P_1x – "*past of* x", or "*past* (what, who) x". F_1x – "*future of* x", or "*future* (what, who) x". Cx – "*change, flow, movement* of x". T_2x – "*termination of* x". C_1x – "*conservation* of x". C_2x – "*self-conservation* of x". S_2x – "*striving* for x". These evaluation-functions are defined by Table 2.

Now let us move from evaluation-functions determined by *one* evaluation-argument to evaluation-functions determined by *two* evaluation-arguments.

Table 3. The evaluation-functions determined by two evaluation-arguments

x	Y	D^2xy	I^2xy	K^2xy	S^2xy	B^2xy	E^2xy	Y^2xy	Z^2xy	W^2xy	L^2xy	F^2xy	V^2xy	T^2xy	C^2xy
g	g	B	g	g	b	G	g	B	b	B	B	b	b	B	g
g	b	B	g	b	g	B	b	G	b	G	B	b	b	B	b
b	g	G	b	b	g	G	b	G	g	G	G	g	b	G	g
b	b	B	g	b	g	G	g	B	b	B	B	b	b	B	g

Glossary for Table 3. (In this paper the upper number-index 2 standing immediately after a capital letter informs that the indexed letter stands for a function determined by *two* arguments.) D^2xy – "difference, distinguishability of y from x", or "y's being *discernible (different)* from x". I^2xy – "indistinguishability (nonbeing of difference) of y from x", or "y's being indiscernible (indistinguishable) from x". K^2xy – "*unity (oneness)* of x and y", or "*joint being* of x and y", or "x's and y's *being together*". S^2xy – "*separation, division* of x and y". B^2xy – "*being (existence) of y in x*", or "y's being contained in x". E^2xy – "*equivalence, identity (coincidence)* of x and y". Y^2xy – "*difference between x* and y". Z^2xy – "y's *contradiction to (with)* x". W^2xy – "*contradiction (opposition) between x* and y". L^2xy – "*limiting x* by y". F^2x – "y's (own) frontier, border for x". V^2xy – "*verge*, divide, frontier, borderline (dead streak) between x and y". T^2xy – "transcending, crossing, overcoming, going beyond, cutting across x *by* y". C^2xy – "y's *being an axiological consequence* of x", or "y's *axiological following* from x". These evaluation-functions determined by two evaluation-arguments are defined by Table 3.

For perfect defining the algebraic system of metaphysics as formal axiology, now it is necessary to give exact definitions of its basic concepts, namely: "*formal-axiological equivalence*"; "*law of metaphysics*" (or, which is the same, "*formal-axiological law*"); "*formal-axiological contradiction*"; "*formal-axiological consequence (entailment)*". These fundamental metaphysical concepts are precisely defined as follows.

Definition DF-1 of the binary relation called "*formal-axiological-equivalence*": in the algebraic system of metaphysics as formal axiology, any evaluation-functions Ξ and Θ are *formally-axiologically equivalent* (this is represented by the expression "Ξ=+=Θ"), if and only if they acquire identical values from the set {g (good), b (bad)}, under any possible combination of the values of their variables.

Definition DF-2 of the notion "law of metaphysics" (or, which is the same, "*formal-axiological law*"): in the algebraic system under consideration, any evaluation-function Θ is called "*formally-axiologically (or necessarily, or universally, or absolutely) good* one", or a "*law of metaphysics*" (or a "*law of algebra of formal axiology*"), if and only if Θ acquires the value *g (good)* under any possible combination of the values of its evaluation-variables. In other words, the function Θ is *formally-axiologically (or constantly, or absolutely) good* one, iff Θ=+=g (good).

Definition DF-3 of the notion "*formal-axiological contradiction*": in two-valued algebra of metaphysics as formal axiology, any evaluation-function Θ is called "*formally-axiologically (or invariantly, or absolutely) bad* one", or a "*formal-axiological contradiction*", if and only if Θ acquires the value *b (bad)* under any possible combination of the values of its evaluation-variables. In other words, the function Θ is called a "*formal-axiological contradiction*", or a "*formally-axiologically (or necessarily, or universally, or absolutely) bad* evaluation-function", iff Θ=+=b (bad).

Definition DF-4 of the binary relation called "formal-axiological-consequence (entailment)": in the two-valued algebraic system of metaphysics (as formal axiology), an evaluation-function Θ formally-axiologically follows from an evaluation-function Ξ, iff it is impossible that Ξ has the value g and Θ has the value b. In other words, Θ is a formal-axiological consequence of Ξ, iff $C^2\Xi\Theta$ is a positive-constant-evaluation-function (a formal-axiological law) in that algebraic system, i.e. iff $C^2\Xi\Theta$=+=g (good).

Along with the above-exploited *tabular* definitions of evaluation-functions, one can utilize also *analytic* definitions of evaluation-functions by means of relevant formal-axiological equivalences (equations of the algebraic system of metaphysics), for instance, our main object of investigation in this paper – Leibniz's principle of identity of indiscernibles is *analytically* defined in the algebraic system by the following formal-axiological equation DF-5.

Definition DF-5: E^2xy=+=$K^2I^2xyI^2yx$, where the symbol E^2xy stands for "*identity* of *x* and *y*".

From the artificial language of mathematical model, this equation may be translated into the natural language by the sentence: "Identity of *x* and *y* is uniting nonbeing of difference of *y* from *x* with nonbeing of difference of *x* from *y*". As identity and difference are opposites, the evaluation-function called "difference between" may be defined analytically as follows.

Definition DF-6: Y^2xy=+=$NK^2I^2xyI^2yx$, where the symbol Y^2xy stands for "*difference between x* and *y*".

Graphic Modeling the Above-Formulated System by Formal-Axiological Square and Hexagon

Sometimes visual images of geometric models are effective means of/for clarifying complicated systems of logical relations among abstract notions. For example, an adequate visualization (graphic model) of logical structure of I. Kant's complicated epistemology system has been invented by J.-Y. Béziau.[5] For the first time in history of philosophy, Béziau has represented Kant's doctrine of synthetic *a-priori* knowledge by means of a hexagon modeling logical relations among the notions "analytic", "synthetic", "*a priori*", "*a posteriori*". In my opinion, the geometric model invented by Béziau is perfectly adequate to its original. It is helpful for developing Kant studies further. In this paper I apply the *precedent* made by Béziau to an essentially *analogous* case of Leibniz studies. Namely, below I undertake a hitherto never realized attempt to use hexagon for visual modeling *formal-axiological* aspect of Leibniz's principle of identity of indescenibles. I think that the system of *formal-axiological* relations among the evaluation-functions relevant to this principle may be modeled by Fig 1 and Fig 2.

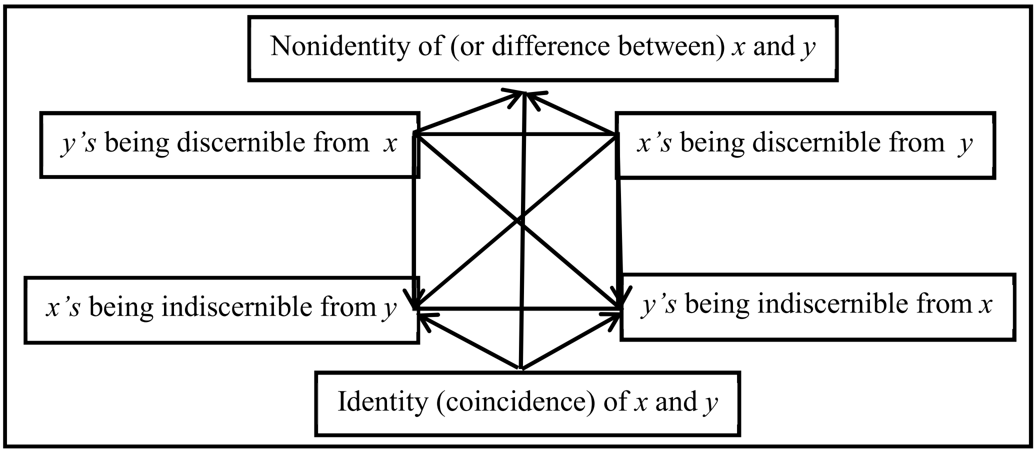

Fig. 1. Formal-Axiological Square and Hexagon Modeling the Idea of Identity of Indiscernibles

Substituting corresponding symbols of the above-introduced artificial language for the natural language expressions used in Fig. 1, results in the following Fig. 2.

5 J.-Y. Béziau: "A Hexagon to Clarify Kant's Two Dualities: Analytic/ Synthetic – A Priori/ A Posteriori",in: Lorenz Demey/ Dany Jaspers/ Hans Smessaert (eds.): *Handbook of the 7th World Congress on the Square of Opposition*, Leuven, Belgium 2022, p. 23; J.-Y. Béziau: "The Power of the Hexagon"; A. Aberdin: "Nelson's Hexagon",in: Lorenz Demey/Dany Jaspers/H. Smessaert (eds.): *Handbook of the 7th World Congress on the Square of Opposition*, pp. 17–19.

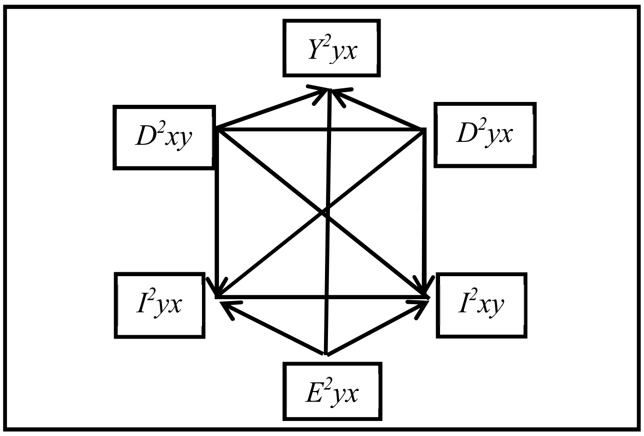

Fig. 2. Formal-Axiological Identity of Indiscernibles

The *formal-axiological contradictoriness* is represented (modeled graphically) in Fig. 1 and Fig. 2, by the lines crossing the square. While utilizing Fig. 2, it is worth keeping in mind that, according to the above-given definitions, in the algebraic system of formal axiology, the binary operations D^2xy and I^2xy are *not commutative*, but the binary operations W^2xy, Y^2xy, and E^2xy are commutative.

The *formal-axiological contradictoriness* is modeled *analytically* by the following equations of the algebraic system of metaphysics.

1) $E^2I^2xyND^2xy$=+=g. For testing the equation see Table 3, Table 1, and definitions DF-1 and DF-2.
2) $E^2I^2yxND^2yx$=+=g. For examining the equation see the same tables and definitions.
3) $E^2Y^2yxNE^2yx$=+=g. For testing the equation see the same tables and definitions.
4) $W^2D^2xyI^2xy$=+=g. For examining the equation see Table 3 and definitions DF-1 and DF-2.
5) $W^2D^2yxI^2yx$=+=g. For examining the equation see Table 3 and definitions DF-1 and DF-2.
6) $W^2E^2yxY^2yx$=+=g. For testing the equation see Table 3 and definitions DF-1 and DF-2.

The *formal-axiological contrariety* is represented in Fig. 1 and Fig. 2 by the upper horizontal line of the square. Analytically the *contrariety* is modeled by the following *formal-axiological* equation.

7) $K^2D^2xyD^2yx$=+=b. For examining the equation, see Table 3 and definitions DF-1 and DF-3.

The *formal-axiological subcontrariety* is represented in Fig. 1 and 2 by the bottom horizontal line of the square. Analytically the *subcontrariety* is modeled by the following *formal-axiological* equation.

8) $NK^2NI^2xyNI^2yx$=+=g. For testing the equation, see Table 1, Table 3, and definitions DF-1 and DF-2.

The *formal-axiological subalternation (subordination, subjection)* relations are represented in Fig. 1 and Fig. 2 by the vertical lines of the square which lines are vectorized (directed from the upper end to the lower one). The arrows represent *formal-axiological consequence* relations. As to the square of opposition, the *formal-axiological subalternation* relations are modeled analytically by the following formal-axiological equations.

9) $C^2D^2xyI^2yx$=+=g. For examining the equation see Table 3 and definitions DF-1, DF-2, and DF-4.
10) $C^2D^2yxI^2xy$=+=g. For testing the equation see Table 3 and definitions DF-1, DF-2, and DF-4.

Now, having created a visual image (graphic model) of/for the above-given system of definitions necessary for adequate understanding the paper, let us start generating *formal-axiological equations* making up a discrete mathematical model of the *formal-axiological* aspect of philosophical ontology in general and of metaphysics of time in particular (taking into an account that being essentially connected with the above-considered theme of identity, philosophical ontology of present, past, and future is directly related to the main theme of 11th International Leibniz Congress).

A System of Formal-Axiological Equations Modeling Metaphysics of Being and of Tenses: Past, Future, and Present

"Le present est plein de l'avenir, et chargé du passé".

It is an obvious linguistic fact that natural human language is very ambiguous. Therefore, it is not surprising that the natural-language statement used as an epigraph for this part of the paper is puzzling (enigmatic); its meaning is not quite clear. Which meaning: *descriptive-indicative* or *formal-axiological* one is implied here? The question is quite relevant but not simple. The proper *formal-logical* interrelations among the *descriptive-indicative* meanings of the terms "Past", "Future", and "Present" are systematically investigated in contemporary symbolic *modal* logic called *temporal* logic.[6] On the contrary, *formal-axiological* interrelations among the *evaluation-functional* meanings of the terms are still missed due to their being still not recognized as such. To fill in the blank, in this paper I am to concentrate on exactly *formal-axiological* aspect of the theme to be investigated below by means of its discrete mathematical model – system of *formal-axiological* equations of the *algebraic system of metaphysics.* Now let us begin generating the list of equations and of their translations from the artificial language into the natural human one which translations are located to the right after every equation immediately after the colon. The artificially created unhabitual sign "=+=" is translated into natural human language by the word "is" which is gravely ambiguous: it has several

6 V. Goranko/A. Rumberg: "Temporal Logic", in: Edward N. Zalta (ed.): *The Stanford Encyclopedia of Philosophy* (Summer 2022 Edition), https://plato.stanford.edu/archives/sum2022/entries/logic-temporal/

qualitatively different meanings in natural human language.[7] (The formal-axiological meaning of "=+=" is defined above by DF-1. The formal-axiological meanings of other artificial symbols are defined above by relevant evaluation-tables.)

11) V^2xy=+=$K^2L^2xyL^2yx$: *borderline (dead streak)* between *x* and *y* is oneness (unity) of limiting *x* by *y* and limiting *y* by *x*.
12) $K^2L^2xyL^2yx$=+=b: oneness (unity) of limiting *x* by *y* and limiting *y* by *x* is a formal-axiological contradiction.
13) V^2xy=+=b: *borderline (dead streak)* between *x* and *y* is a formal-axiological contradiction.
14) $V^2P_1xF_1x$=+=g: *borderline (dead streak)* between past of *x* and future of *x* is a formal-axiological contradiction.
15) $K^2P_1xF_1x$=+=b: oneness (unity) of past and future (their being together) is a formal-axiological contradiction.
16) $E^2P_1xF_1x$=+=b: identity between past and future is a formal-axiological contradiction.
17) $Y^2P_1xF_1x$=+=g: difference between past and future is a law of metaphysics.
18) $S^2P_1xF_1x$=+=g: separation (division) of past of *x* and future of *x* is a law of metaphysics.
19) $TV^2P_1xF_1x$=+=g: *transcending (crossing) borderline* between past of *x* and future of *x* is a law of metaphysics.
20) $CV^2P_1xF_1x$=+=g: *flow (change) of borderline* between past of *x* and future of *x* is a law of metaphysics.
21) Zx=+=$K^2P_1xF_1x$: *present x* is *formally-axiologically equivalent* to unity (oneness) of future of *x* and past of *x*. In this equation and hereafter in this paper, the symbol *Zx* (hitherto not used above) stands for the evaluation-function "*present x*", or "*present time* of *x*".

Thus, the newly introduced symbol *Zx* is included into the alphabet of artificial language of the discrete mathematical model of metaphysics under investigation in the given paper, and formal-axiological meaning of the symbol *Zx* is precisely defined analytically in this model by means of the equation. Let us exploit equation 23) as such a precise *analytic definition* of the evaluation-function "*present x*" in the algebraic system of metaphysics, which definition is, probably, an adequate explication of a hitherto not recognized *formal-axiological meaning* of the statement by Leibniz about present. Certainly, the unexpected *formal-axiological* interpretation of Leibniz' statement and the suggested *analytic definition* of the evaluation-function "*present x*" make up a hypothesis. In my opinion, the hypothesis is truthlike and worth investigating by hypothetic-deductive method. Therefore, in this paper, the hypothesis is accepted as such and the hypothetic formal-axiological meaning of the statement by Leibniz about present is analyzed systematically. From the above-given definitions it follows that the below-placed equivalences are true.

22) $K^2P_1xF_1x$=+=b: unity of future of *x* and past of *x* is formal-axiological contradiction.
23) Zx=+=b: present *x* is formal-axiological contradiction.

7 V. Lobovikov: "The Trinity Triangle and the Homonymy of the Word 'Is' in Natural Language",in: *Philosophy Study* 5/7 (2015), pp. 327–341, https://doi.org/10.17265/2159-5313/2015.07.001.

24) $Zx{=}{+}{=}V^2P_1xF_1x$: present x is borderline between past x and future x.
25) $Bx{=}{+}{=}B^2P_1xF_1x$: being of x is being of future x in past of x.
26) $Bx{=}{+}{=}B^2P_1xZx$: being of x is being of present x in past of x.
27) $Bx{=}{+}{=}B^2M_1WxZx$: being of x is being of present x in material world of x.
28) $B^2M_1WxP_1x{=}{+}{=}$g: being of past of x in material world of x is a law of metaphysics.
29) $E^2xx{=}{+}{=}$g: self-identity of x is a law of metaphysics. (By the way, this equation is a mathematical representation of not the well-known proper logical but a hitherto still not-well-recognized *exactly metaphysical* "law of identity" originally formulated vaguely in the gravely ambiguous natural language by Aristotle.)
30) $NE^2xx{=}{+}{=}$b: nonbeing of self-identity of x is formal-axiological contradiction.
31) $S_2E^2xx{=}{+}{=}$g: striving for self-identity of x is a law of metaphysics.
32) $Bx{=}{+}{=}B^2M_1WxNE^2yy$: being of x is being of self-non-identity of y in the material world of x.
33) $Zx{=}{+}{=}T_1NE^2xx$: present time of x is time of nonbeing of self-identity of x.
34) $C_1Zx{=}{+}{=}$b: conservation of any present x is formal-axiological contradiction.
35) $T_2Zx{=}{+}{=}$b: termination of any present x is a law of metaphysics.
36) $FZx{=}{+}{=}$g: finiteness, definiteness, temporality of present x is a law of metaphysics.
37) $B^2Zxx{=}{+}{=}$g: being of x in present time of x is a law of metaphysics.
38) $B^2ZxF_1x{=}{+}{=}$g: being of future x in present x is a law of metaphysics.
39) $B^2ZxP_1x{=}{+}{=}$g: being of past x in present x is a law of metaphysics. (In my opinion, it is a verisimilar hypothesis that this formal-axiological equation together with the previous one, make up a discrete mathematical model of the statement by Leibniz about present.)
40) $B^2ZT_1xAx{=}{+}{=}$g: being of absolute time of x in present time of x is law of metaphysics.
41) $B^2P_1T_1xAx{=}{+}{=}$g: being of absolute time of x in past time of x is a law of metaphysics.
42) $B^2F_1T_1xAx{=}{+}{=}$g: being of absolute time of x in future time of x is a law of metaphysics.
43) $BZx{=}{+}{=}B^2AxZx$: being of present of x is being of present of x in absolute time of x.
44) $BP_1x{=}{+}{=}B^2AxP_1x$: being of past of x is being of past of x in absolute time of x.
45) $BF_1x{=}{+}{=}B^2AxF_1x$: being of future of x is being of future of x in absolute time of x.
46) $Ax{=}{+}{=}T_1Gx$: absolute time of x is time of God of x.

Graphic modeling formal axiology of tenses as evaluation-functions of the algebraic system may be accomplished by means of the following *formal-axiological* opposition square and hexagon.

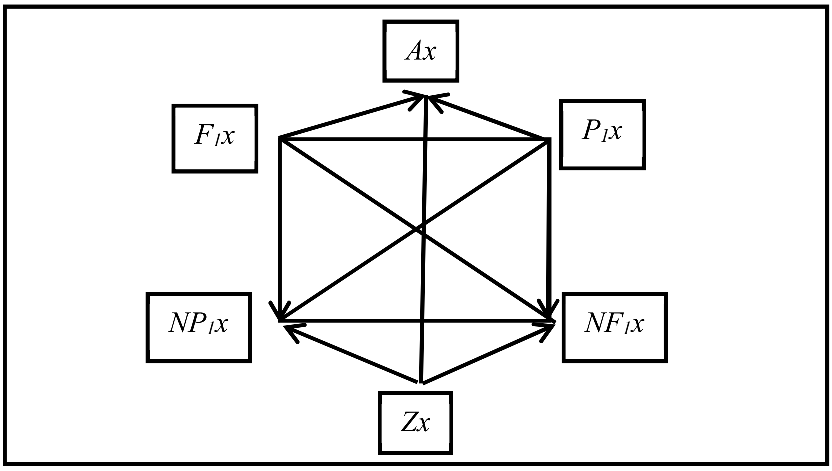

Fig. 3. Formal-Axiological Hexagon of Tenses

G.W. Leibniz' Doctrine of Facts as Contingent Truths, G.E. Moore's Doctrine of Naturalistic Fallacies in Ethics, and D. Hume's Problem of Logical Bridging the Gap between "Is" and "Ought" (Precisely Formulating such a Principle of Mutual Formal-Logical Independence of Facts and Corresponding Contingent Values which is Universal for Empirical Knowledge Exclusively)

While examining the above-listed formal-axiological equations and their translations into natural human language, one must keep in mind the following significant linguistic fact. In the dreadfully ambiguous natural human language, usually, the *formal-axiological equivalence* relation "=+=" is represented by the puzzling (gravely ambiguous) words "is", "means", "implies", "entails", "equivalence". Each of these words *has substantially different semantic meanings*. As in the usual language of humans, the mentioned semantically ambiguous words may stand for the proper logic connectives (or binary logic relations) called "equivalence" and "entailment", there is a serious danger of encountering hard logic-linguistic confusions (psychologically natural *illusions* of "mortal" paradoxes) generated by manifestly prohibited substituting for each other the substantially different concepts, namely, the *formal-axiological* relation "=+=" and the proper formal-logic operation "equivalence" (or "=+=" and the proper formal-logic operation "implication"). Such chaotic mixing and substituting are manifestly forbidden in the algebraic system of formal axiology. Ignoring this prohibition leads to allegedly mortal paradoxes. Now having made the warnings at the level of natural human language, let us formulate them more precisely (by means of the artificial language) as the following *fact/value-separation* rule: from (α =+= ω) it does not follow logically that ($\Phi\alpha \leftrightarrow \Phi\omega$); from ($\Phi\alpha \leftrightarrow \Phi\omega$) it does not follow logically that (α =+= ω). In this formulation of the rule, the symbols α and ω represent any evaluation-functions, and the symbols $\Phi\alpha$ and $\Phi\omega$ stand for either true or false statements of *fact*, which statements are affirming that, *in fact*, α and ω are realized (take place in actual world).

Certainly, all statements of *fact* are statements of *being*, but not every statement of *being* is statement of *fact*. Statements of *necessary* being are not statements of *fact* because, according to G.W. Leibniz, statement of *fact* is a statement of *not necessary* but *contingent* being.[8] In XX century, Leibniz' doctrine of *facts* as true statements of exactly *contingent* being has been accepted and developed further by R. Carnap.[9] In this paper, and in my previous publications on the theme[10], while discussing interrelations among facts, values, and norms, I use the term "fact" in exactly that meaning which has been defined by Leibniz.

The above-provided exact formulation of the obvious scientific methodology principle of mutual formal-logical independence of corresponding *facts* (*contingent* truths) and r*elative* values (*empirical* assessments) is too abstract, perhaps. Therefore, now it is quite relevant to show this general scientific methodology principle at work in quite a concrete particular case.

Let the symbol "(t < +10◦C)" denote the evaluation-function "*x*'s human-body temperature is less than 10 degrees above zero centigrade." Also, let the symbol "(t > +70◦C)" denote the evaluation-function "*x*'s human-body temperature is more than 70 degrees above zero centigrade." If all the relevant concrete circumstances of the case under investigation are sufficiently definite and values of the variables *x* and Σ are fixed, then the evaluation-functions (t < +10◦C) and (t > +70◦C) take either good or bad value depending of the value of Σ. In the two-valued algebraic system of formal axiology, for every *x* and Σ, it is obviously true that ((t < +10◦C) =+= (t > +70◦C)). This *formal-axiological* equivalence deals with *empirically* defined *contingent* values exclusively. The equation ((t < +10◦C) =+= (t > +70◦C)) has no *formal-logic-consequence* relations to corresponding *fact*-statements.

According to the above-defined meaning of the symbol Φ necessarily used in the above-provided exact formulation of the fact/value-separation principle under instantiation, the symbol Φ(t > +70◦C) denotes the fact-fixing-judgement "*x*'s human-body temperature is more than 70 degrees above zero centigrade." In its turn, the symbol Φ(t < +10◦C) denotes the fact-fixing-judgement "*x*'s human-body temperature is less than 10 degrees above zero centigrade". Being statements of *facts*, Φ(t > +70◦C) and Φ(t < +10◦C) are either true or false ones. Consequently, it is quite relevant to link the two fact-fixing-statements by the binary logic connective "↔" (biconditional). The complex proposition (Φ(t > +70◦C) ↔ Φ(t < +10◦C)) is a logic linkage of *facts*: this equivalence is not formal-axiological one as it does not deal with moral and other values studied by general axiology. According to the

8 G.W. Leibniz: "Of Universal Science, or Philosophy Calculus [Ob universal'noj nauke, ili filosofskom ischislenii]", in: *G.W. Leibniz. Writings in Four Volumes, V. 3 [G.V. Lejbnic, Sochineniya v chetyrekh tomah, T.3]*, Moscow 1984, pp. 494–500, here p. 496 (in Russian).

9 R. Carnap: *Meaning and Necessity: a study in semantics and modal logic*, Chicago/London 1956.

10 V.O. Lobovikov: "Knowledge Logic and Algebra of Formal Axiology: A Formal Axiomatic Epistemology Theory Sigma Used for Precise Defining the Exotic Condition Under Which Hume-and-Moore Doctrine of Logically Unbridgeable Gap Between Statements of Being and Statements of Value is Falsified", in: *Antinomies* 20/4 (2020), pp. 7–23, https://doi.org/10.24411/2686- 7206-2020-10401.

above-formulated fact/value-separation rule, accomplishing the below-located "formal logic inference" is committing a blunder, as there is no proper formal-logic entailment relation between the "premise" and the "corollary".

$$\frac{(t > +70^\circ C) =+= (t < +10^\circ C)}{\Phi(t > +70^\circ C) \leftrightarrow \Phi(t < +10^\circ C)}$$

In this scheme of logically invalid (fallacious) inference, moving from the formal-axiological "premise" to the alleged formal-logical consequence ("corollary") is strictly prohibited by the above-formulated fact/value-separation rule. Violating this rule heads to logic contradictions with *empirical* knowledge of biology of human body, namely, to formal logic deriving the false purely *factual* (empirically false) "conclusion" from the empirically true purely *evaluative* "premise". Converse moving from the obvious falsity of $(\Phi(t > +70^\circ C) \leftrightarrow \Phi(t < +10^\circ C))$ to alleged falsity of $((t > +70^\circ C) =+= (t < +10^\circ C))$ by the "*modus tollens*" is also forbidden by the fact/value-separation rule, because there is no formal-logic-consequence relation between the two qualitatively different formal equivalences. Thus, the empirically grounded general fact/value-separation rule is exemplified.

Today, almost all specialists in natural *sciences* (physics, biology) and in the *empirical* humanities (positivistic sociology, ethics, jurisprudence) believe that the above-formulated fact/value-separation rule is a universal law. However, in my opinion, there is a very important nontrivial question concerning the rule, namely, is domain of the rule's relevant applicability somehow confined (or is it absolutely unlimited)? I have manifestly formulated and systematically investigated such a challenging hypothesis according to which the domain of relevant applicability of the fact/value-separation rule is limited; universality of the rule is not absolute but relative; the rule is actually universal in relation *to empirical* knowledge (of *facts*) exclusively.

Consequently, in principle, there is a surprising possibility of perfect formal-logical bridging the gap between statements of being and corresponding ones of value. However, the surprising possibility does not contradict logically to/with the *empirically* grounded general fact/value-separation principle because the wonderful possibility exists *beyond* the *limited* domain of relevant applicability of the fact/value-separation rule, namely, in the *not empty* realm of *not empirical* but *a-priori* knowledge of *not contingent* but *necessary* being. The paradigm-breaking hypothesis has been tested by the hypothetic-deductive method. Such a logically formalized axiomatic epistemology system called "Sigma" has been invented[11], in which a "mole-hole" for perfect formal-logical inferring statements of being from corresponding ones of value has been discovered.[12] Here the "mole-hole" denotes a challenging theorem of actual possibility of perfectly logical bridging the "gap" between statements of *necessary* being and statements of *necessary* value, which

11 V. O. Lobovikov: "A Logically Formalized Axiomatic Epistemology System $\Sigma + C$ and Philosophical Grounding Mathematics as a Self-Sufficing System", in: *Mathematics* 9/16 (2021), https://doi.org/10.3390/math9161859.

12 V. O. Lobovikov: "Knowledge Logic and Algebra of Formal Axiology".

theorem has been precisely formulated and formally proved in the formal axiomatic theory Sigma. The wonderful theorem has been applied to philosophical foundations of physics.[13]

13 V.O. Lobovikov: “A Formal Deductive Inference of the Law of Inertia in a Logically Formalized Axiomatic Epistemology System Sigma from the Assumption of Knowledge A-Priori-Ness”,in: *Journal of Applied Mathematics and Physics* 9/3 (2021), pp. 441–467, https://doi.org/10.4236/jamp. 2021.93031; V. O. Lobovikov: “Formal Inferring the Law of Conservation of Energy from Assuming A-Priori-ness of Knowledge in a Formal Axiomatic Epistemology System Sigma”,in: *Journal of Applied Mathematics and Physics* 9/5 (2021), pp. 1011–1040, https://doi.org/10.4236/jamp. 2021.95070; V. O. Lobovikov: “Formally Deriving the Third Newton’s Law from a Pair of Nontrivial Assumptions in a Formal Axiomatic Theory ‘Sigma-V’”, in: *Journal of Applied Mathematics and Physics* 10 (2022), pp. 1561–1586, https://doi.org/10.4236/jamp. 2022.105109; V. O. Lobovikov: “Formally Inferring Galileo Galilei Principle of Relativity of Motion in an Axiomatic System ‘Sigma+V’ from a Triple of Nontrivial Assumptions”,in: *Journal of Applied Mathematics and Physics* 10 (2022), pp. 2459–2498, https://doi.org/10.4236/jamp. 2022.108167.